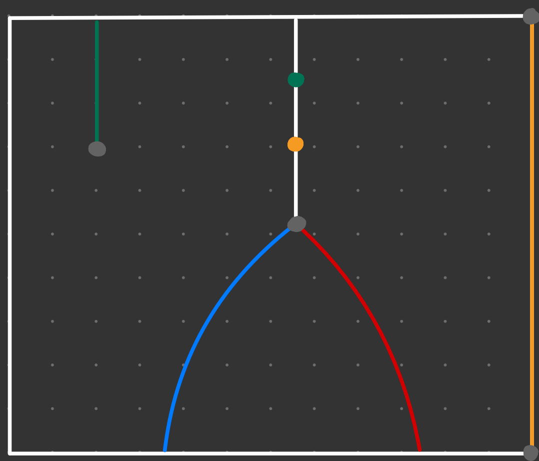
Computing with Tube Categories

Slides & Papers :

jcbridgeman.bitbucket.io



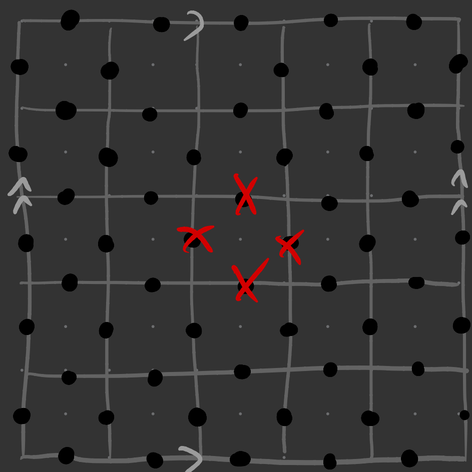
Work with D. Barter

C. Jones

A. Hahn , T. Osborne, R. Wolf

Toric Code

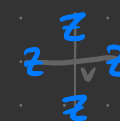
$$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$$



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X^2 = Z^2 = 1 \qquad XZ = -ZX$$

$$H = -\sum_f$$ 

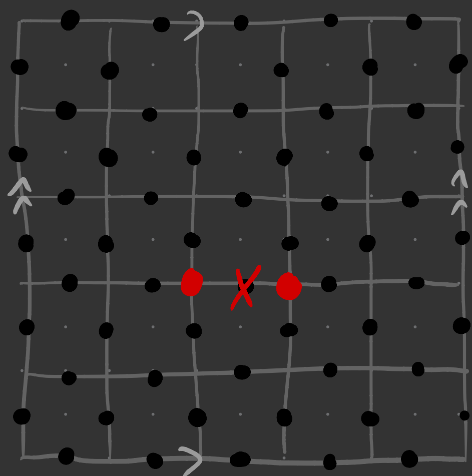
$$-\sum_v$$ 

Ground state $= |\quad\rangle + |\circ\quad\rangle + |\quad\circ\rangle + |\circ\quad\circ\rangle + \cdots$

$= |0\rangle^{\otimes N}$

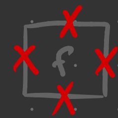
Toric Code

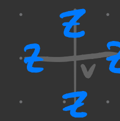
$$\bullet = \mathbb{C}^2 = \text{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$$

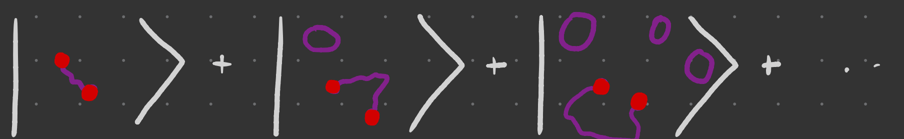


$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

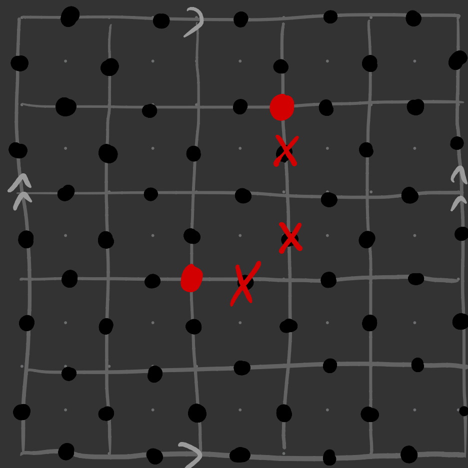
$$X^2 = Z^2 = 1 \qquad XZ = -ZX$$

$$H = -\sum_f$$ 

$$-\sum_v$$ 

Excited state : 

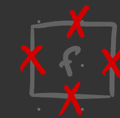
Toric Code

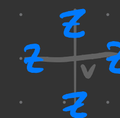


$$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$$

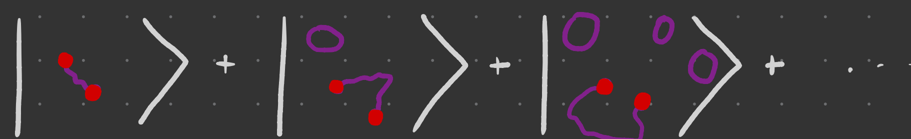
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X^2 = Z^2 = 1 \qquad XZ = -ZX$$

$H = -\sum_f$ 

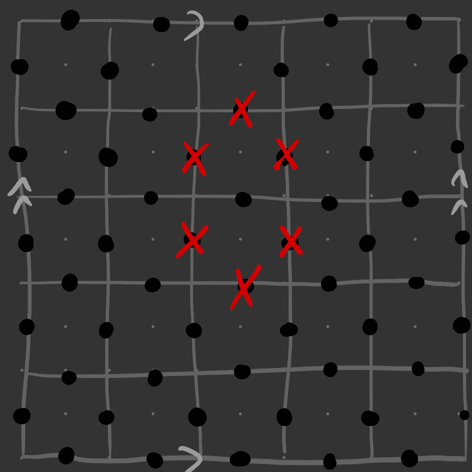
$-\sum_v$ 

Excited state :



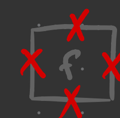
Toric Code

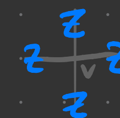
$$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$$



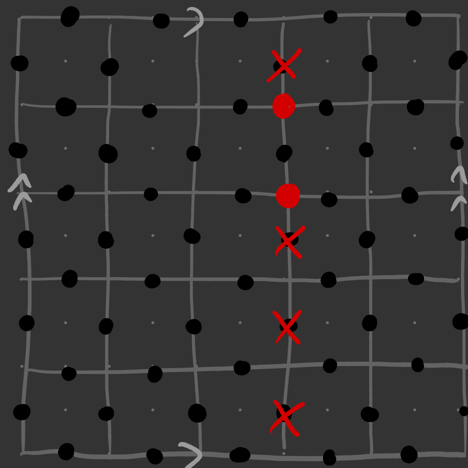
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X^2 = Z^2 = 1 \qquad XZ = -ZX$$

$H = -\sum_f$ 

$- \sum_v$ 

Toric Code

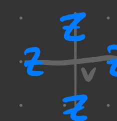


$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$

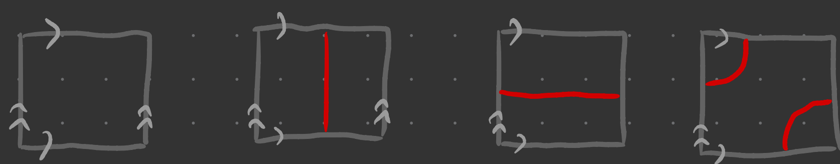
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$X^2 = Z^2 = 1 \qquad XZ = -ZX$

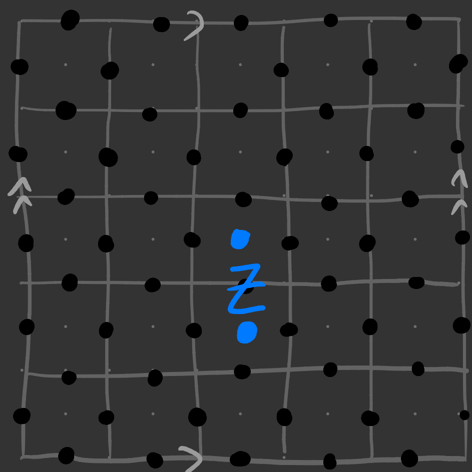
$H = -\sum_f$ 

$- \sum_v$ 

Ground states :



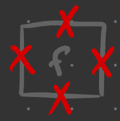
Toric Code

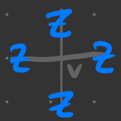


$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\qquad$ $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

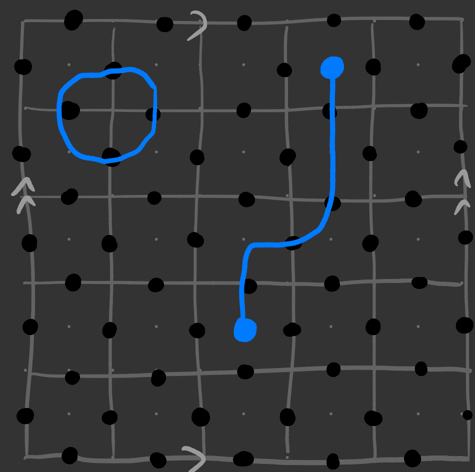
$X^2 = Z^2 = 1$ $\qquad$ $XZ = -ZX$

$H = -\sum_f$ 

$-\sum_v$ 

Toric Code

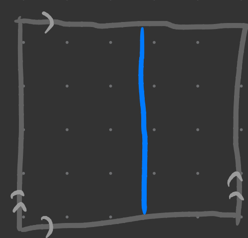
$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$



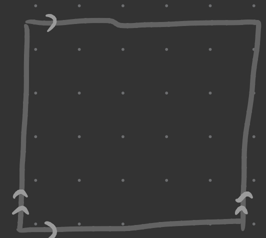
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

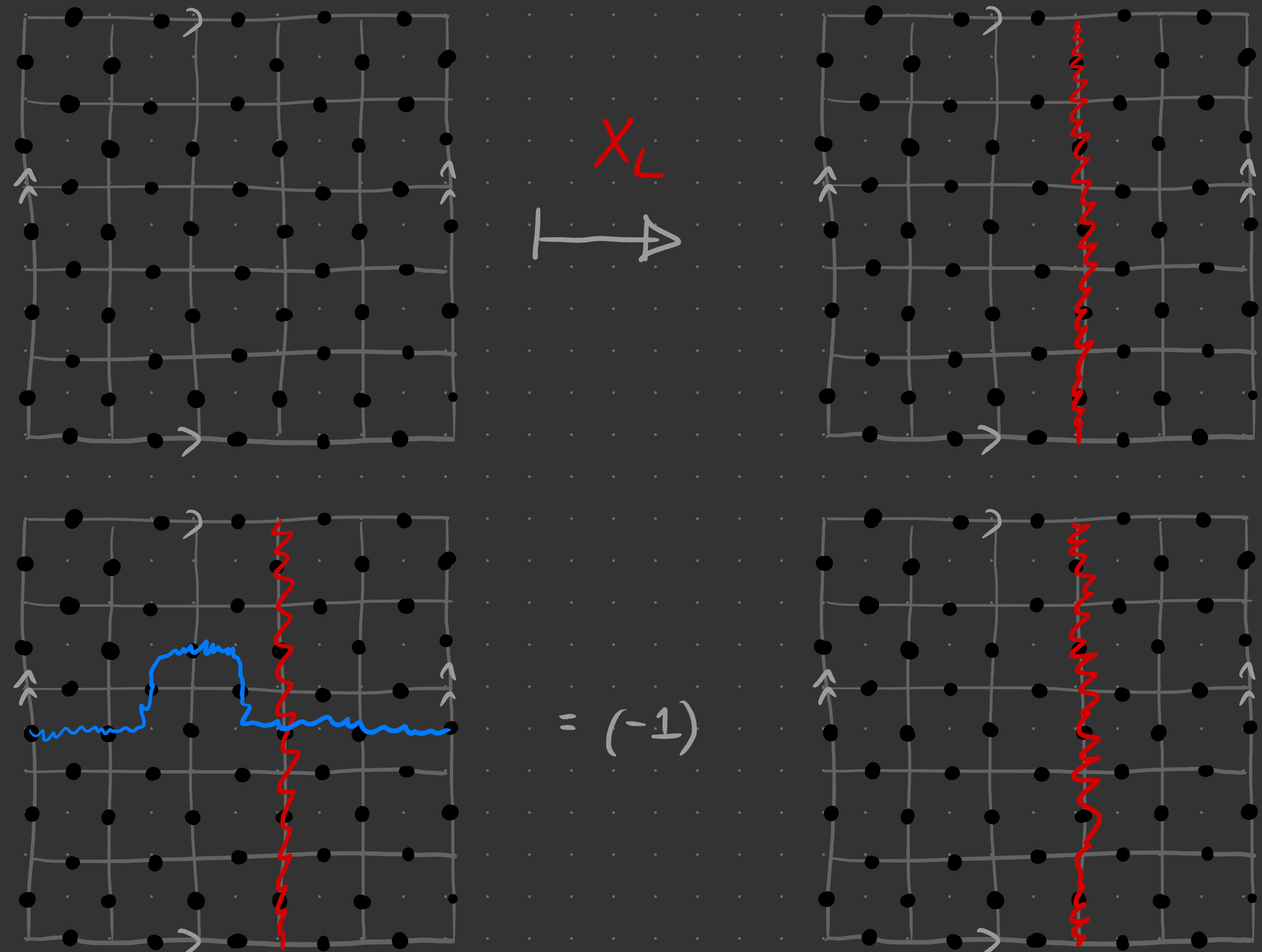
$X^2 = Z^2 = 1 \qquad XZ = -ZX$

$$H = -\sum_f \prod_{e \in \partial f} X_e - \sum_v \prod_{e \ni v} Z_e$$



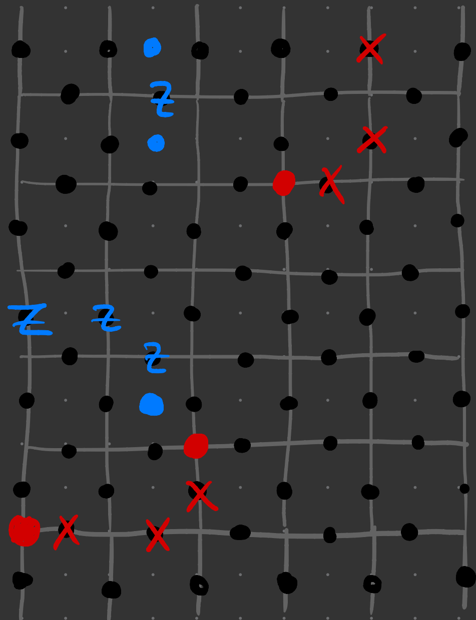
$=$




X_L
$= (-1)$

Toric Code with boundaries

$$\bullet = \mathbb{C}^2 = \mathrm{Span}_{\mathbb{C}}\{|0\rangle, |1\rangle\}$$



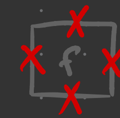
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

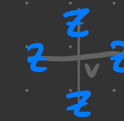
$$X^2 = Z^2 = 1 \qquad XZ = -ZX$$

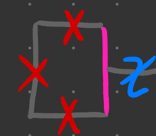
$$H = -\sum_f \prod_{e \in f} X_e - \sum_v \prod_{e \ni v} Z_e - \sum X X X - \sum Z Z Z$$

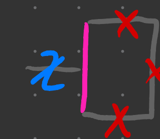
Ground states :

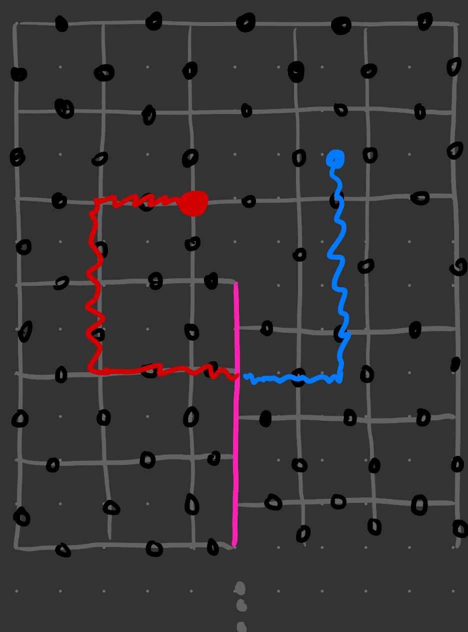
Toric Code with defects

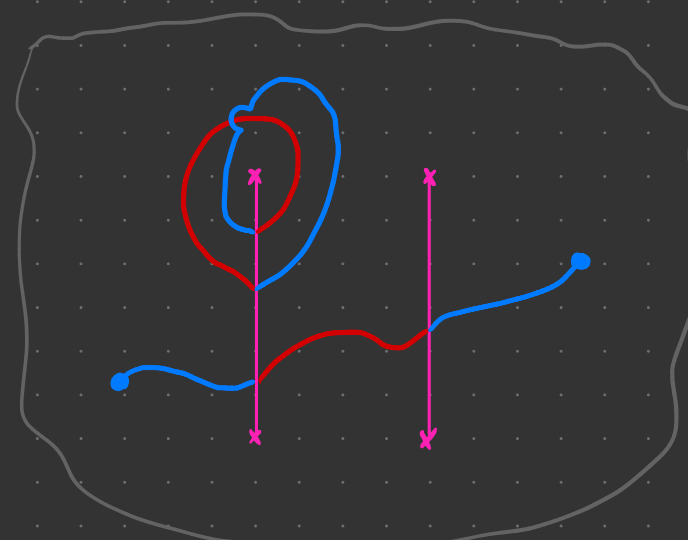
$H = -\sum_f$ 

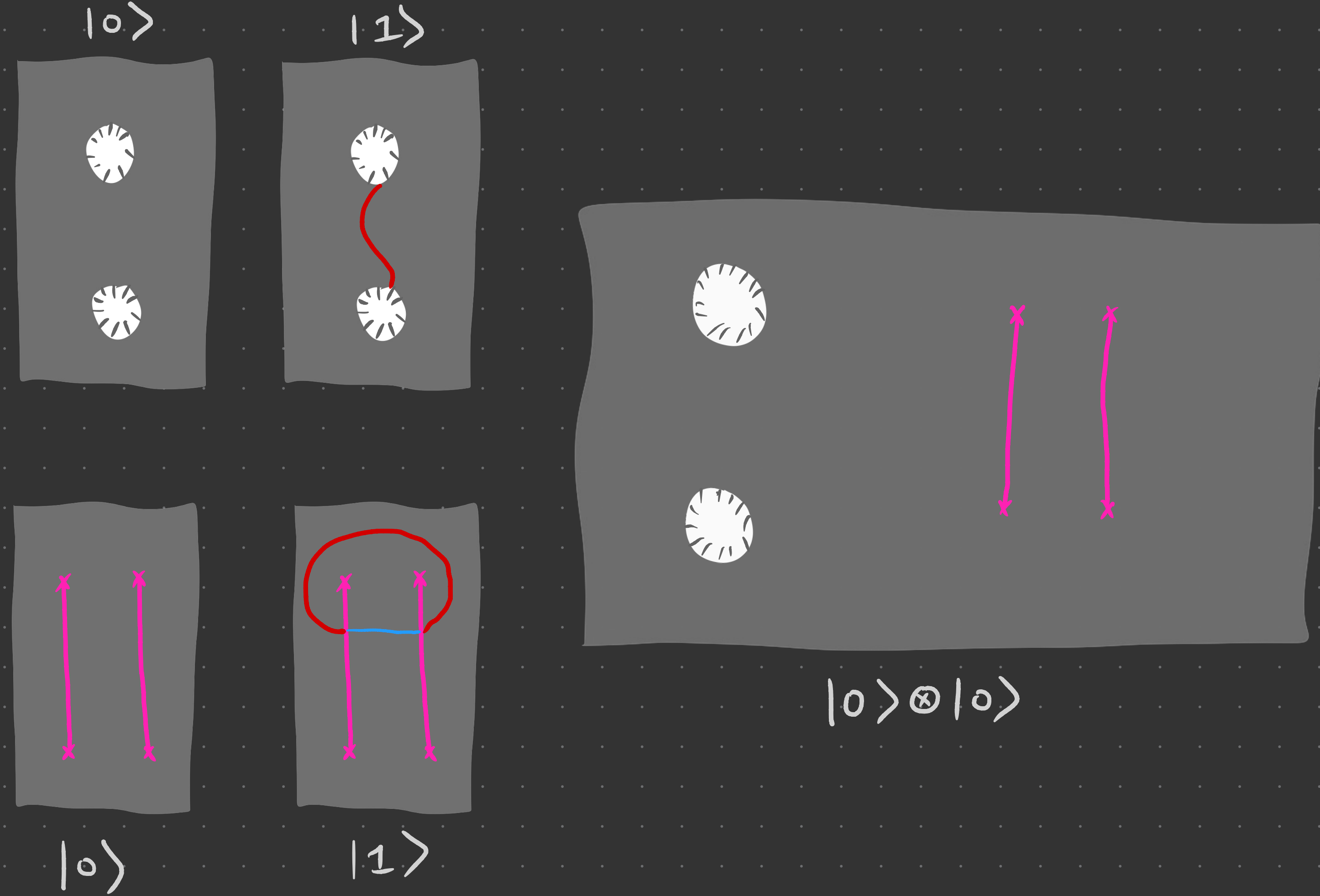
$-\sum_v$ 

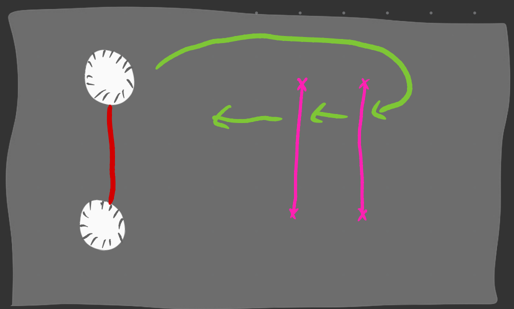
$-\sum$ 

$-\sum$ 

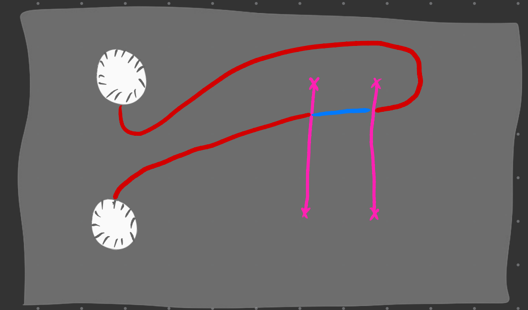





|0⟩
|1⟩
|0⟩
|1⟩
|0⟩⊗|0⟩

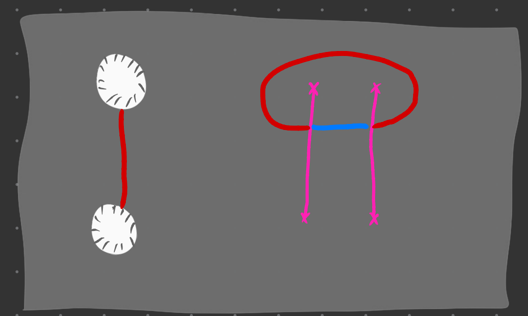


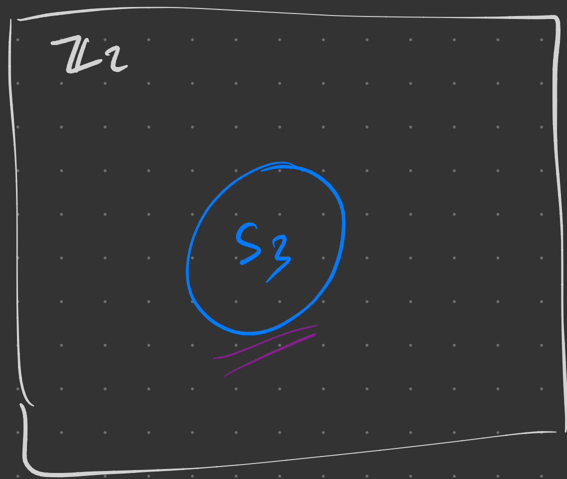
$|10\rangle$

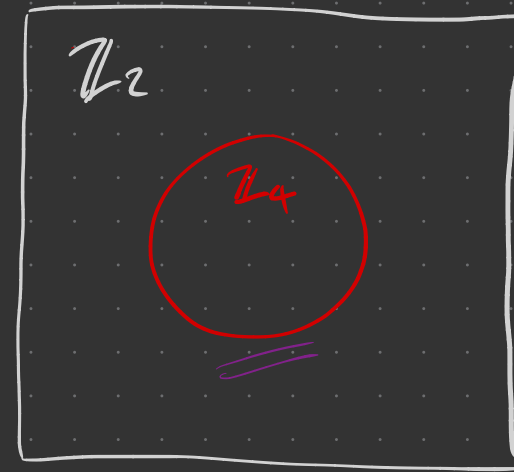


$=$

$|11\rangle$







$$\mathbb{Z}_2 \left| S_3 \right|^{\mathbb{Z}_2} = \quad \mathbb{Z}_2 \quad \mathbb{Z}_2$$

→ Adding defects to topological codes can be useful for quantum computing.

→ To design QC schemes, need to know many properties of the defect theory.

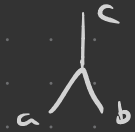
Fusion Cat $\mathcal{C}$

$$\Big\downarrow$$

Lattice model with excitations

$$Z(\mathcal{C})$$

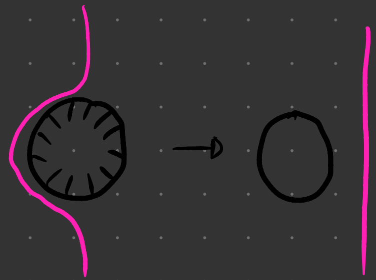
Fusion Cat



Domain Wall / Boundary :

(Bi) Module Cat

Fusing domain walls

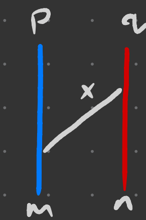


$$\mathcal{M} \underset{\mathcal{C}}{\otimes} \mathcal{N} \cong \mathrm{Kar}\big(\mathcal{L}ad(\mathcal{M},\mathcal{N})\big) \cong \text{Reps of } \mathcal{L}ad(\mathcal{M},\mathcal{N})$$

$\mathcal{L}ad(\mathcal{M},\mathcal{N})$:

objects (m,n)

$\mathrm{Hom}((m,n),(p,q))$:



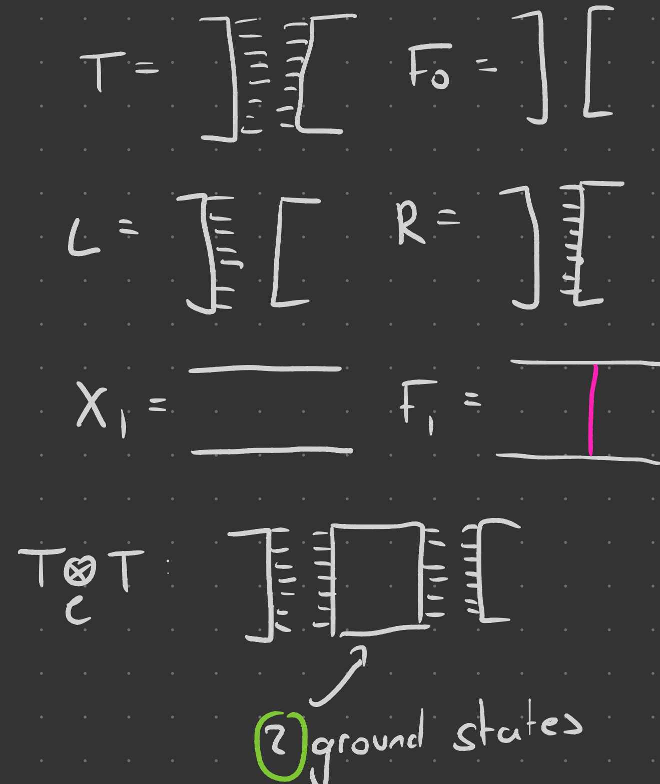
$\mathrm{Kar}(\mathcal{C})$: objects : $(A,\ e : A \to A)$ $e^2 = e$

(classes of) simples in $\mathcal{M} \underset{\mathcal{C}}{\otimes} \mathcal{N}$ $\longleftrightarrow$ Irreducible representations.

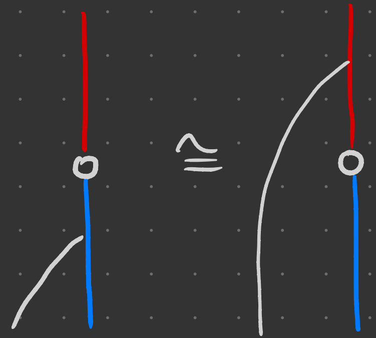
$\mathcal{C} = \mathrm{Vec}\,\mathbb{Z}_2$ (Toric Code)

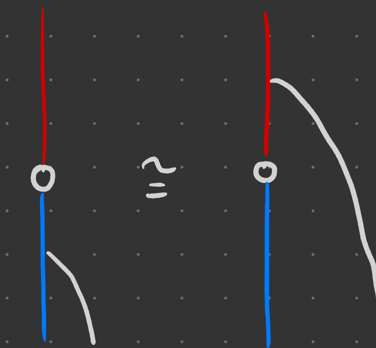
6 bimodules

	T	L	R	F_0	X_1	F_1
T	$2T$	T	$2R$	R	T	R
L	$2L$	L	$2F_0$	F_0	L	F_0
R	T	$2T$	R	$2R$	R	T
F_0	L	$2L$	F_0	$2F_0$	F_0	L
X_1	T	L	R	F_0	X_1	F_1
F_1	L	T	F_0	R	F_1	X_1



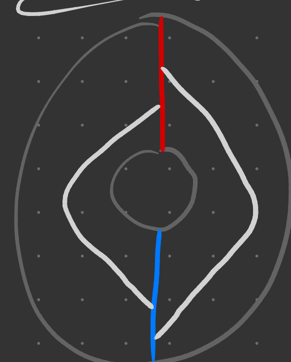
Point defects $\leftrightarrow$ (Bi) module functors.

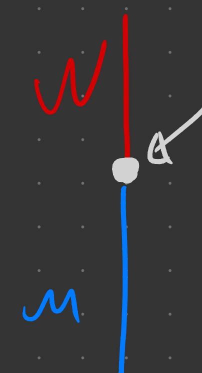




Tube categories

Hom :

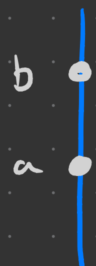




Representations of $\mathrm{Tub}(\mathcal{M}, \mathcal{N})$

Vertical fusion = Functor composition

$$\mathcal{C} \curvearrowright \mathcal{M} \Rightarrow \mathrm{End}_{\mathcal{C}}(\mathcal{M}) \equiv_{ME} \mathcal{C}$$

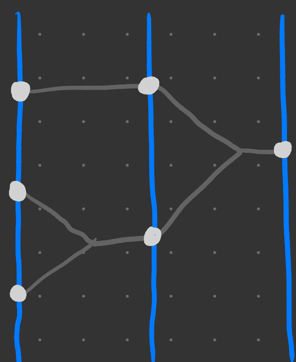


$$\cong \sum_c N_{ab}^{c}$$

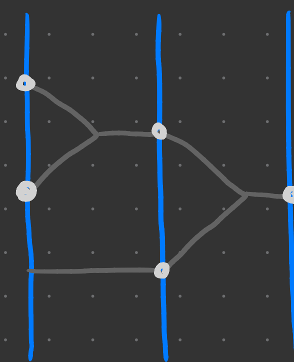
$\mathcal{H}_3 \curvearrowright \mathcal{M}$:

$\mathrm{End}(\mathcal{M})$:

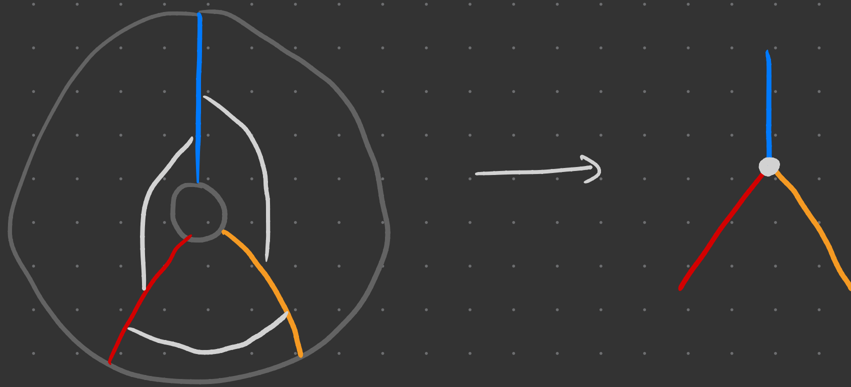
1	ν	η	μ
ν	$1+2\nu+2\eta+\mu$	$2\nu+\eta+\mu$	$\nu+\eta+\mu$
η	$2\nu+\eta+\mu$	$1+\nu+\eta+\mu$	$\nu+\eta$
μ	$\nu+\eta+\mu$	$\nu+\eta$	$1+\nu$

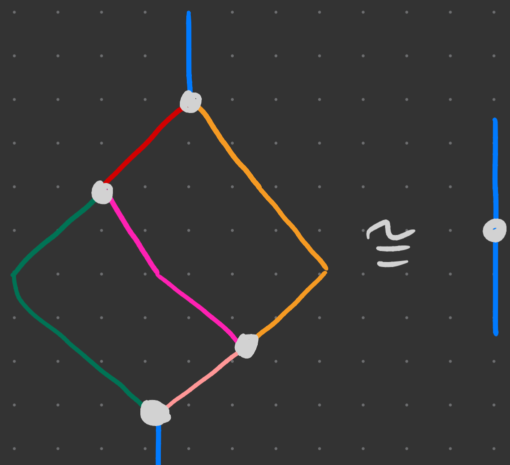


$$\cong$$



Gauging Obstructions





First ENO obstruction O_3

Vanishes for $\mathbb{Z}_p$, S_3

→ Representations of (defect) picture algebras
let you compute many things by manipulating
matrix algebras.

→ Lattice agnostic ⇝ Simplifies understanding what's
possible in a given topo. order

→ Rule out universality for 'simple' input e?

jcbridgeman.bitbucket.io