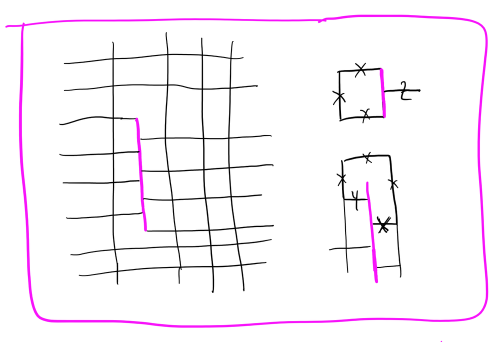
Friday, 18 October 2019 3:36 pm

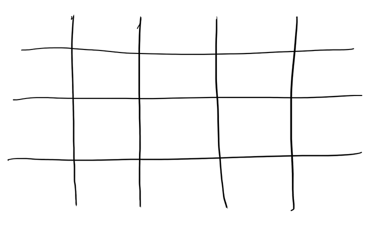
- 1806.01279 Domain walls
- 1810.09469 Point defects
- 1901.08069 Point defects ⟺ Representation thy
- 1907.06692 Associators, Non-abelian, Computational approach.

w D. Barter & C. Jones.

(2+1) Topological Phases with Defects

- Q₁: What fault tolerant gates can we perform with topological codes?
- Q₂: What about if we include defects?
- Q₃: What are defects?





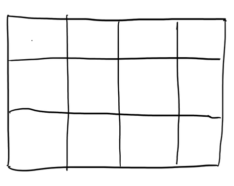
$15 + 16 = 31$ physical

12 stars

20 plaquettes

$\prod$ plaquettes $= 1$.

⇒ No qubits (logical)



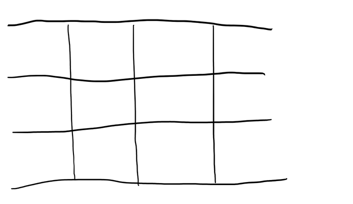
$15 + 16 = 31$ physical

12 plaquettes.

20 stars

$\prod$ stars $= 1$

⇒ No logical qubits



$9 + 16 = 25$ physical

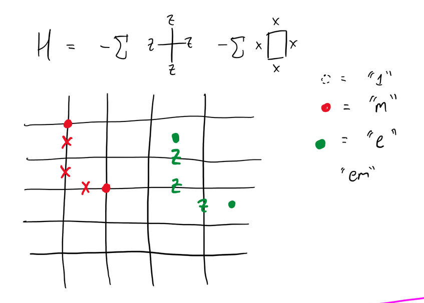
12 plaquettes

12 stars

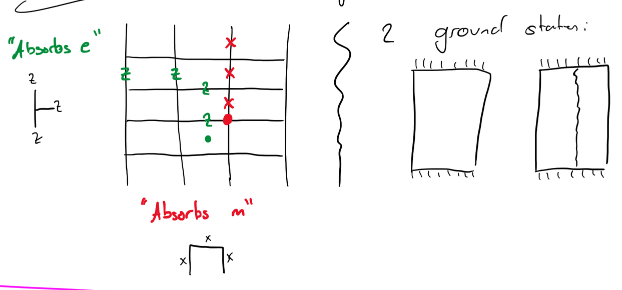
No relations

⇒ 1 logical qubit.

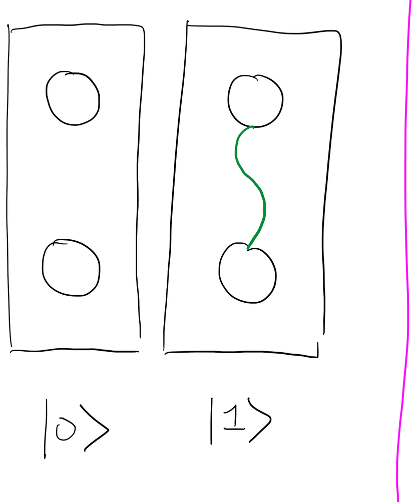
Toric code:



Boundaries 2 kinds "Rough" & "Smooth"

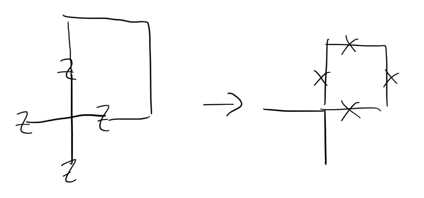


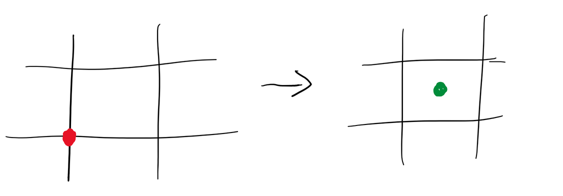
Encoding qubits in defects.

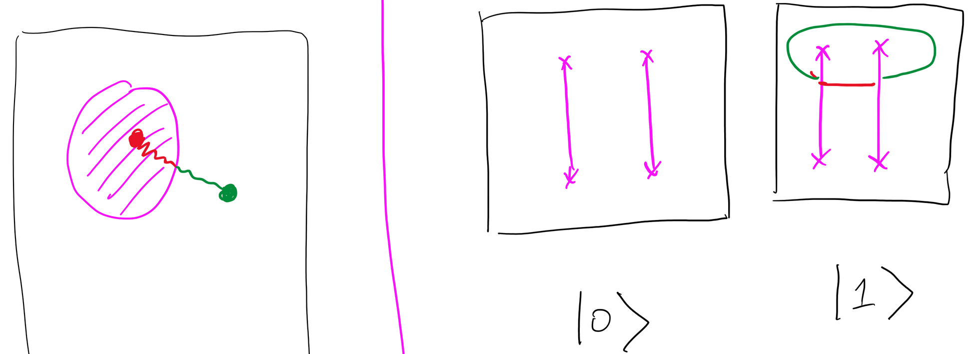


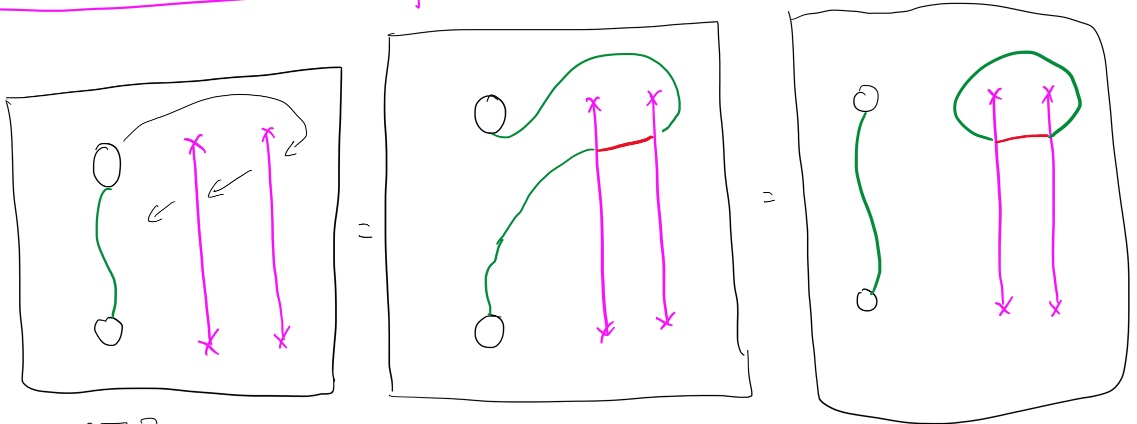
Symmetry

$H^{\otimes N} H_{TC} H^{\otimes N} \sim H_{TC}$.



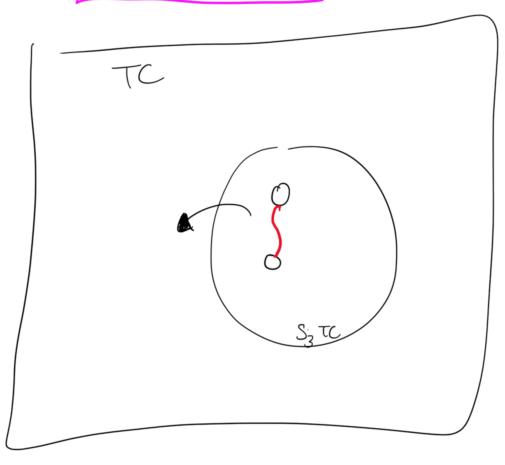






1609.04673 CNOT.

→ 1811.06738



What is going on? More general description.

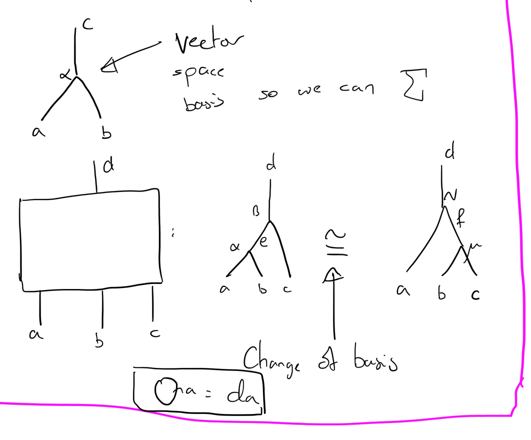
Ground state of toric code.

Closed loops = ground state of vertex terms.

All closed loops = ground state.

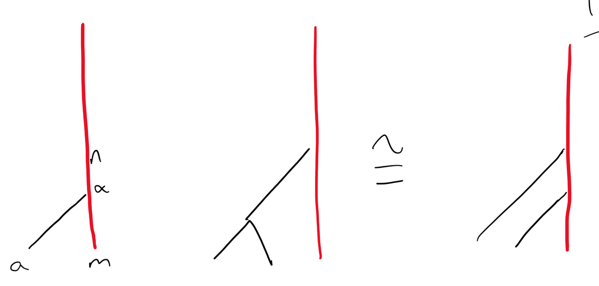
Generalization: String nets.

$\mathcal{C} = \{1, a, b, c, \ldots\}$

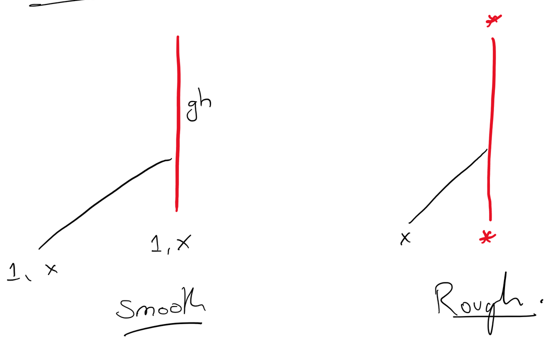


Ground state = Σ pictures.

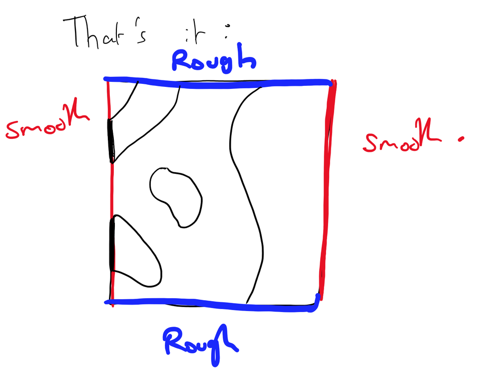
Boundaries? More pictures. Declare these are ground state pictures



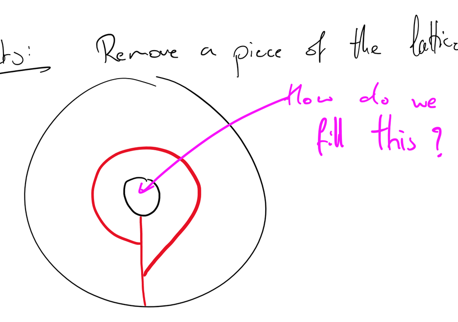
Toric code:



That's it:



Defects: Remove a piece of the lattice.



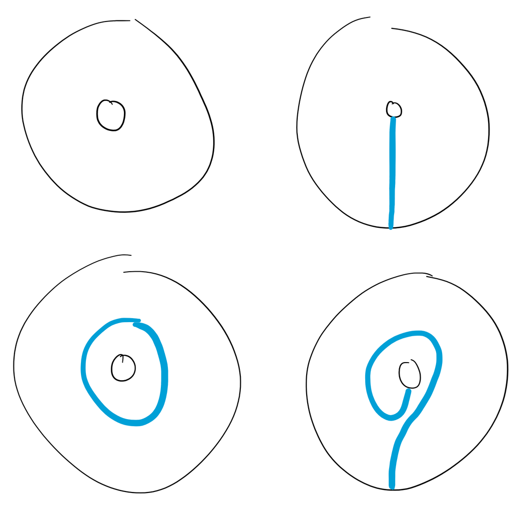
Action of annuli

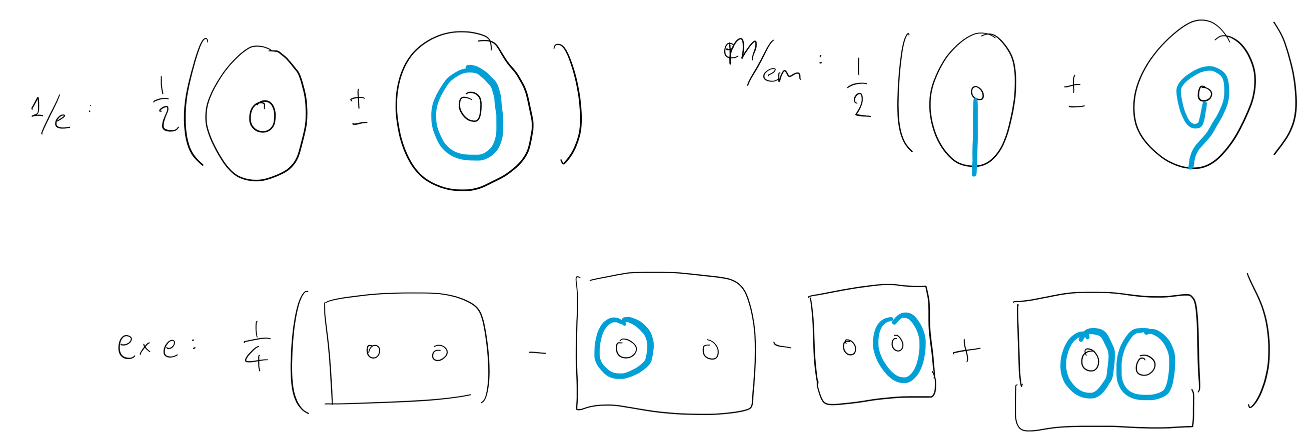
⇓

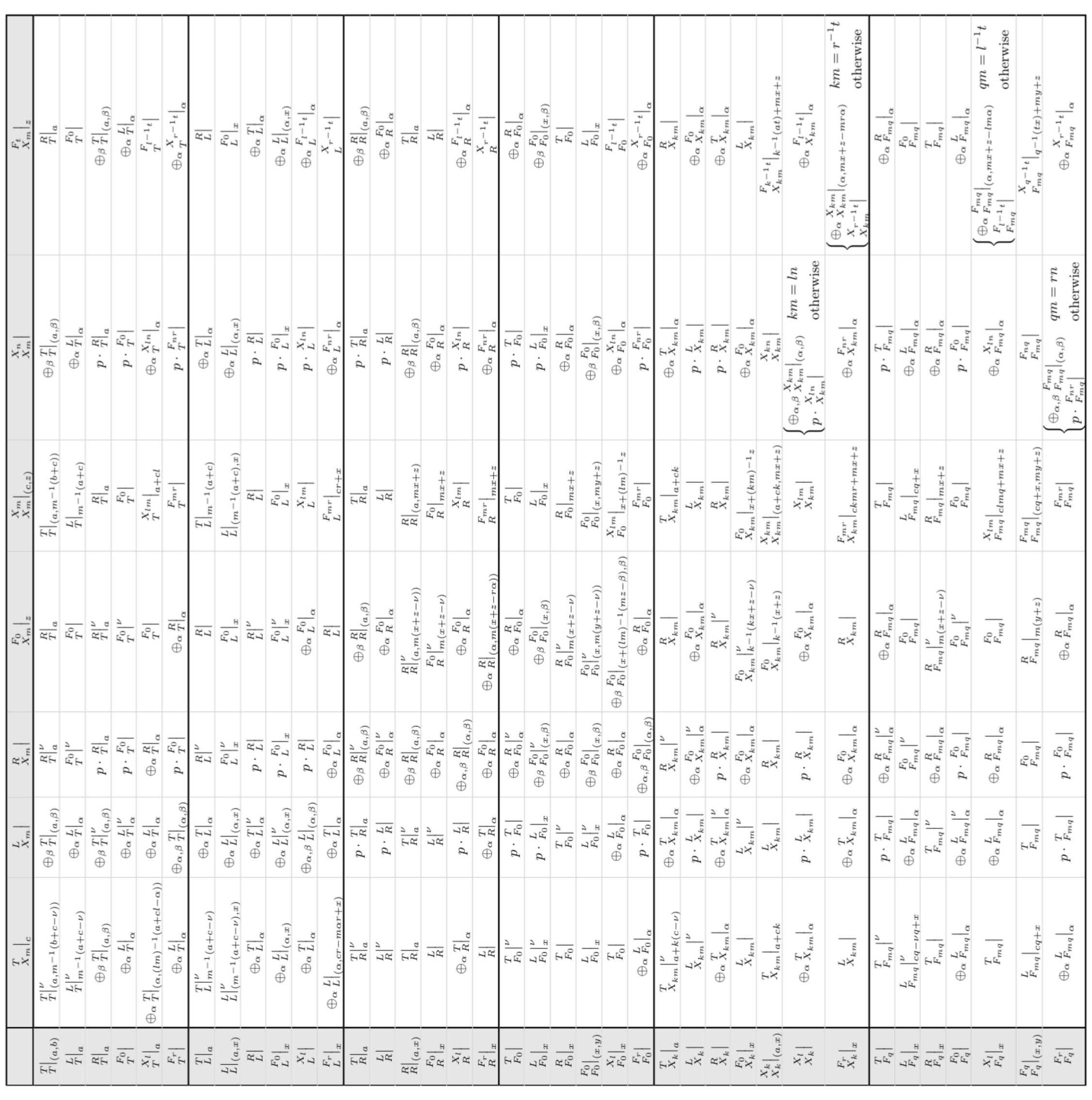
Representation of some algebraic structure.

Artin-Wedderburn thm: Finite ss alg over ℂ ≅ ⊕ Full matrix algebras.

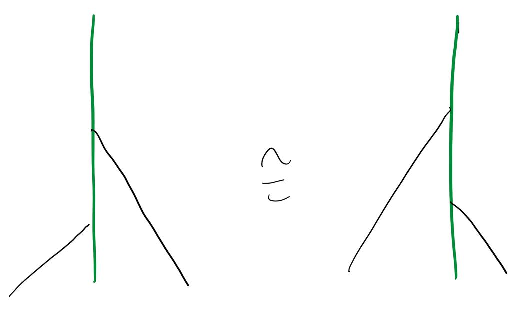
Toric code:

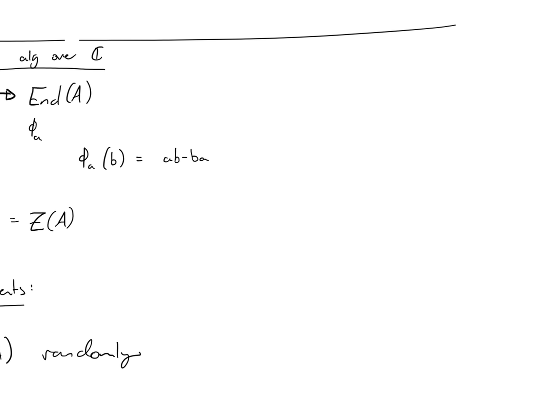






More general computations





- $\phi : A \longrightarrow \mathrm{End}(A)$ (A ss alg over ℂ)

  $a \longmapsto \phi_a$

  $\phi_a(b) = ab - ba$

Kernel of $\phi = Z(A)$

Find central idempotents:

1) Pick $a \in Z(A)$ randomly

$L_a : Z(A) \longrightarrow Z(A)$

$b \longmapsto ab$

Eigenvectors e_i of L_a

$a e_i e_j = \lambda_i e_i e_j = \lambda_j e_i e_j$

⇒ $e_i e_j = 0$ unless $\lambda_i = \lambda_j$ (randomly chosen won't have this)

$e_i^2 = \mu_i e_i \rightsquigarrow$ $\boxed{\tilde{e}_i = \frac{1}{\sqrt{\mu_i}} e_i}$

Now we know that

$B_i = A e_i$ is full matrix algebra.

Decompose $B_i \to M_{N_i \times N_i}(\mathbb{C})$