Ribbon Operators in 2D Topologically Ordered Spin Systems

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ARC Centre of Excellence in Engineered Quantum Systems
The University of Sydney

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ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

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- Understanding quantum matter
  - 2 dimensional topologically ordered models
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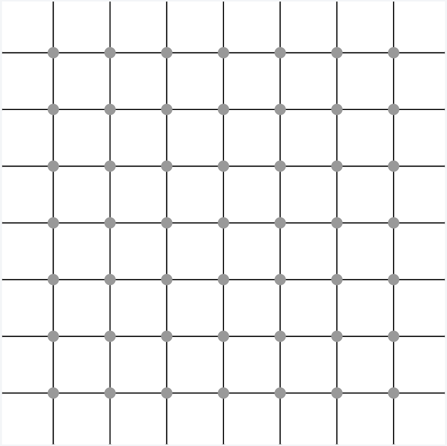
  - No local order parameter, no symmetry
  - New exchange statistics not observed in nature
- Topological codes
  - Quantum information protected against arbitrary errors

Conventional ordering: Ising Model



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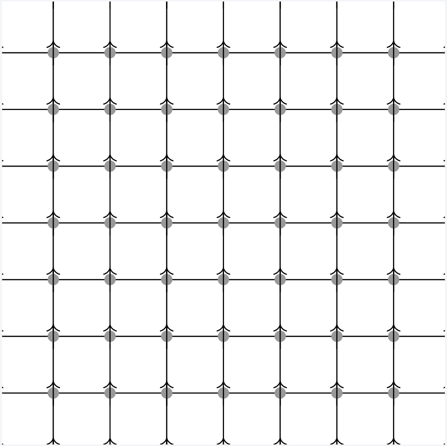

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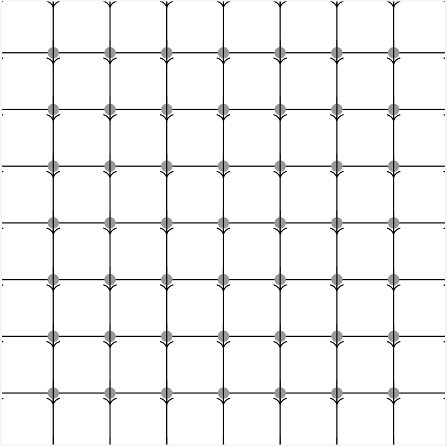

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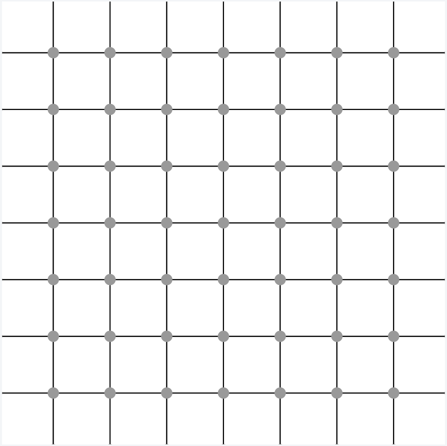

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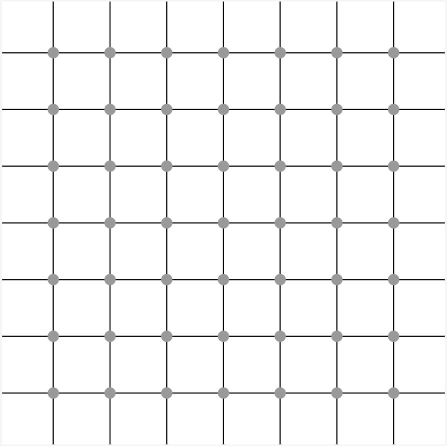

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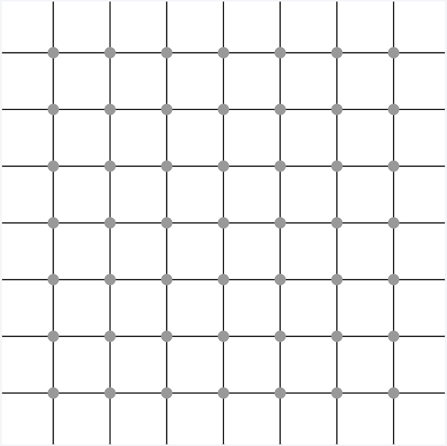

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- Quantum phases: $T \to 0$ limit
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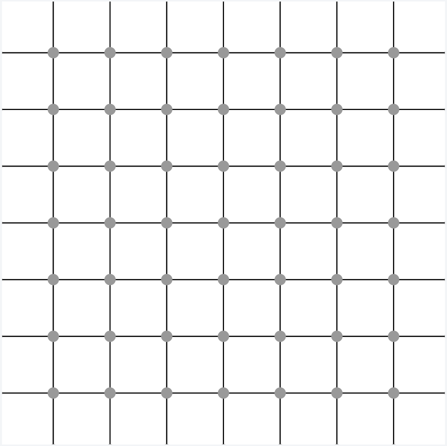
- Magnetisation $\frac{1}{N} \sum \langle Z_j \rangle$
  - Can detect ordering with local measurement

Ising Model: Excitations



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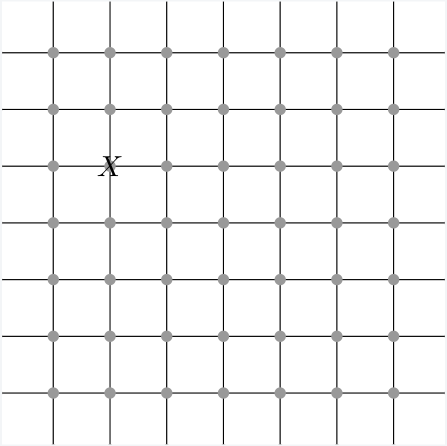

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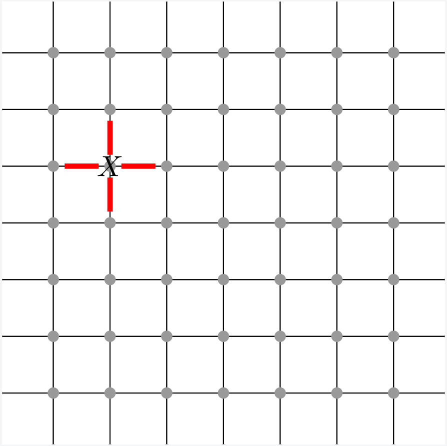

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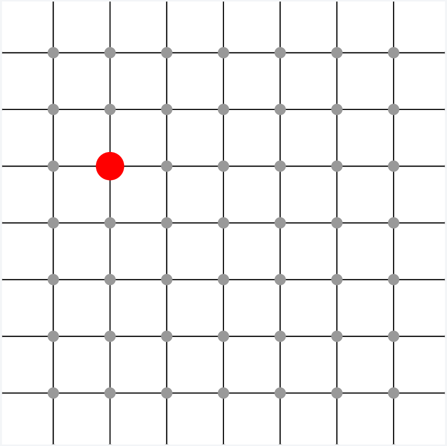

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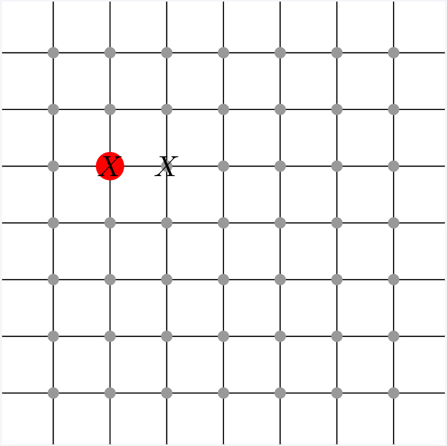

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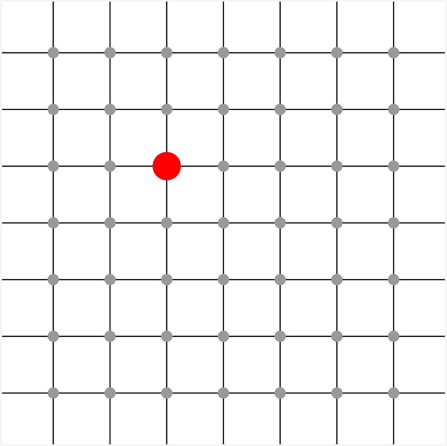

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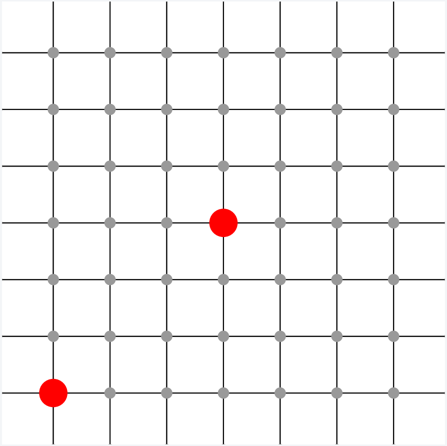

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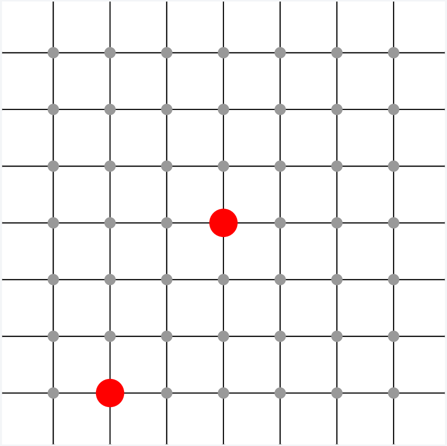

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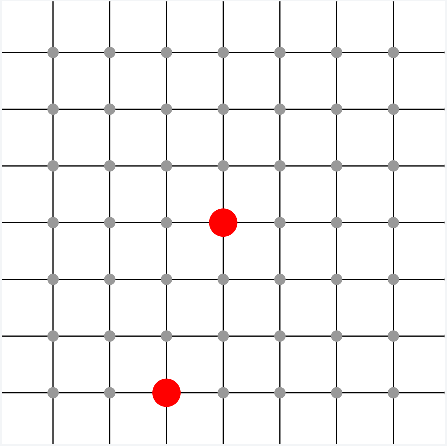

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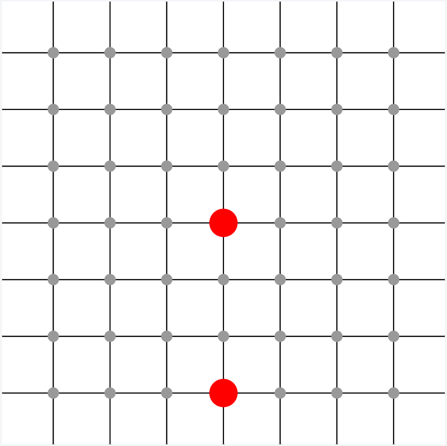

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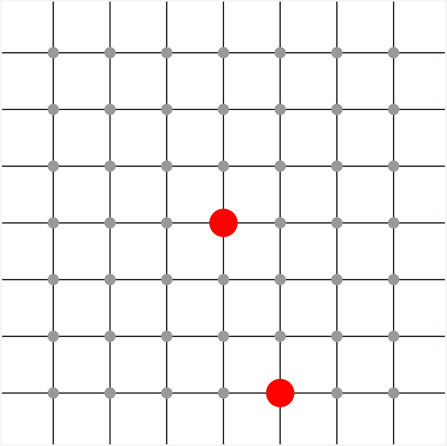

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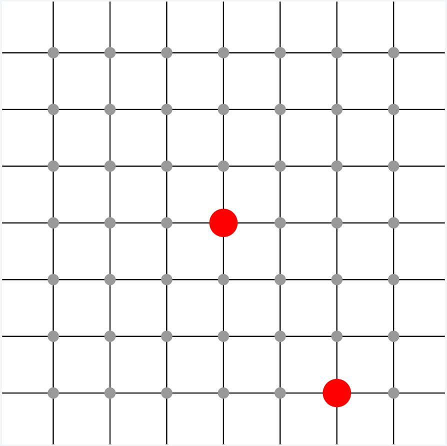

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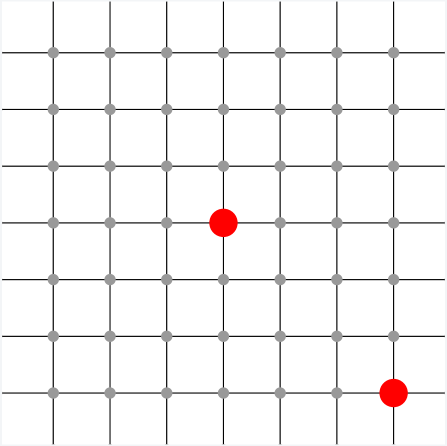

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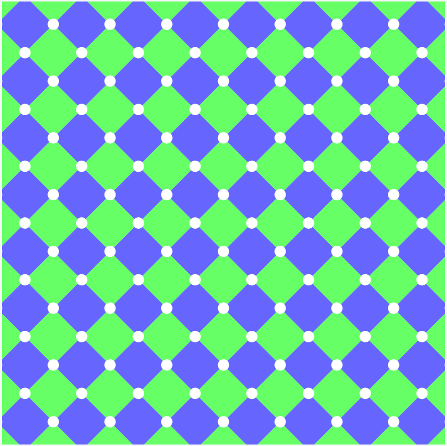
A Simple Example: $\mathbb{Z}_2$ Toric Code



$$H = -\sum_{\text{blue}} \begin{array}{ccc} & X & \\ X & & X \\ & X & \end{array} - \sum_{\text{green}} \begin{array}{ccc} & Z & \\ Z & & Z \\ & Z & \end{array}$$

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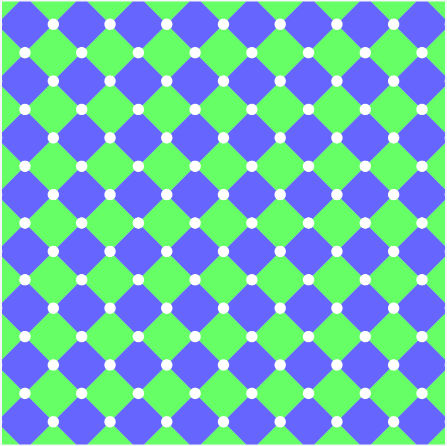

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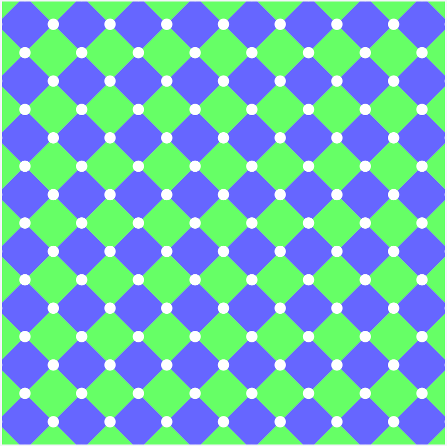

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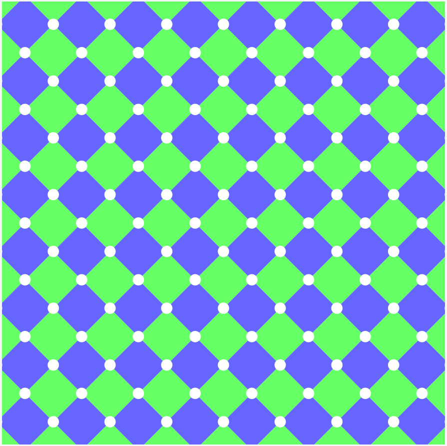

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- Supports many of the characteristic properties of topological ordered models

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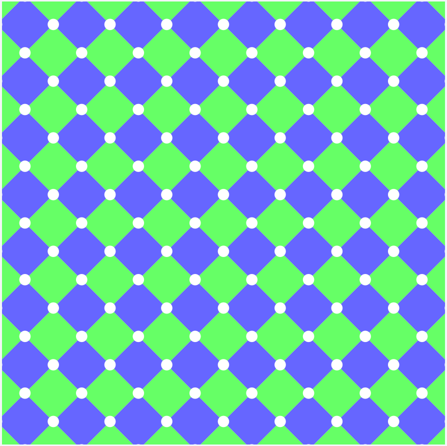

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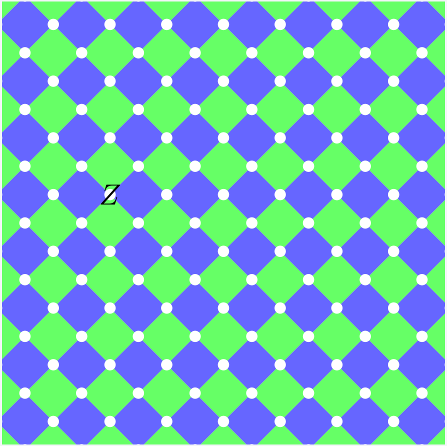

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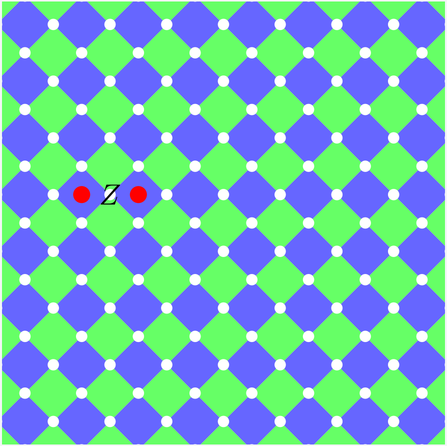

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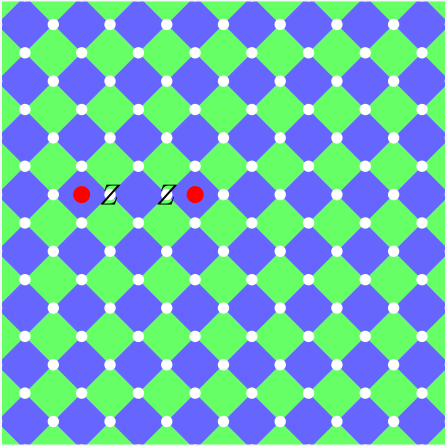

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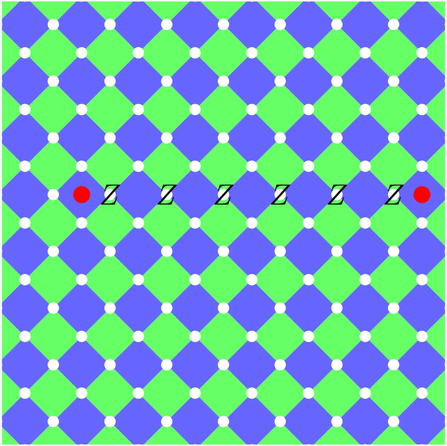

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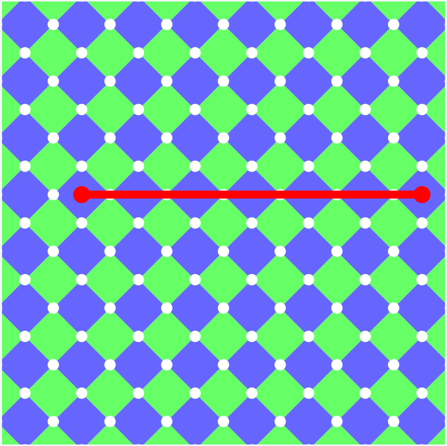

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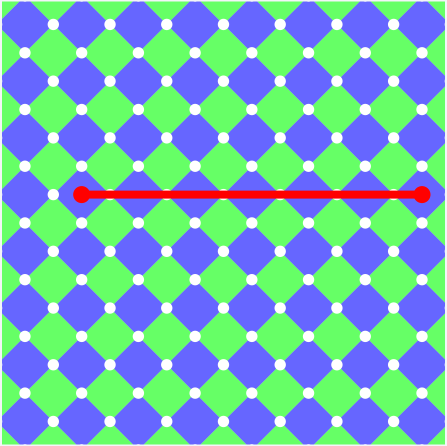

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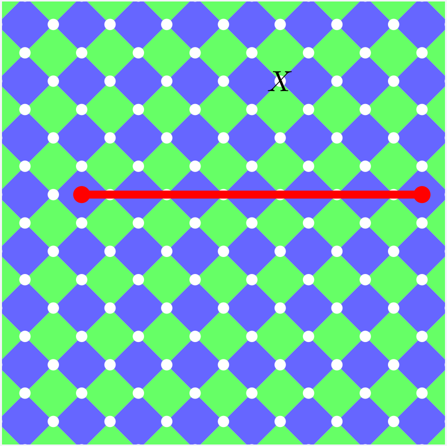

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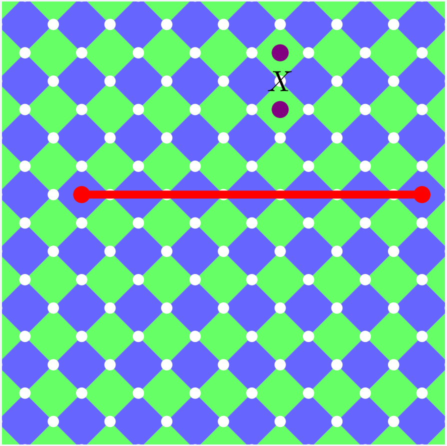

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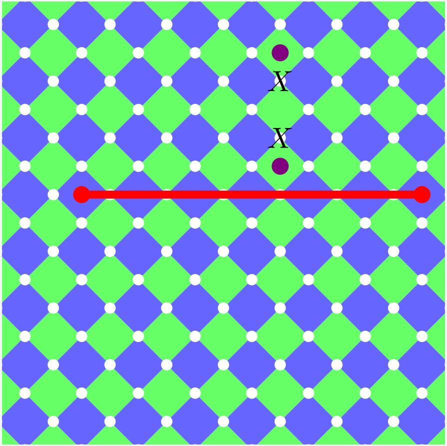

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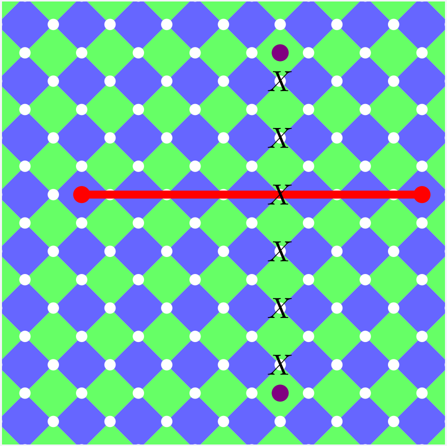

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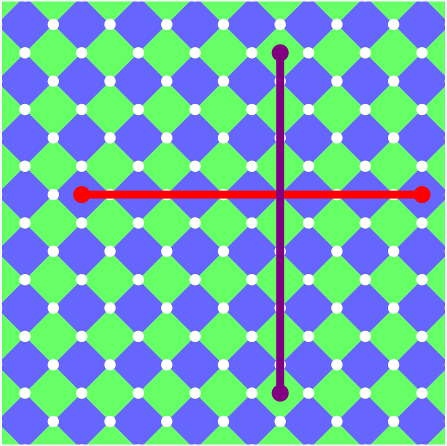

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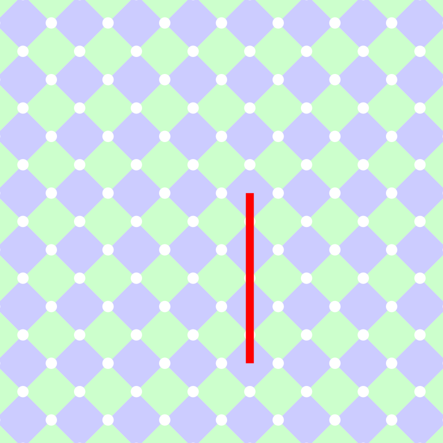

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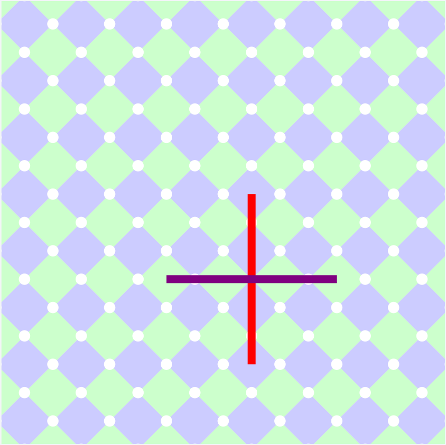

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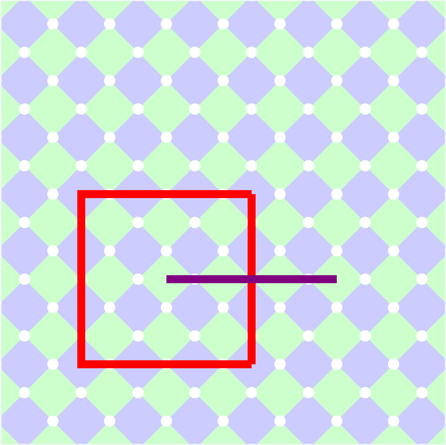

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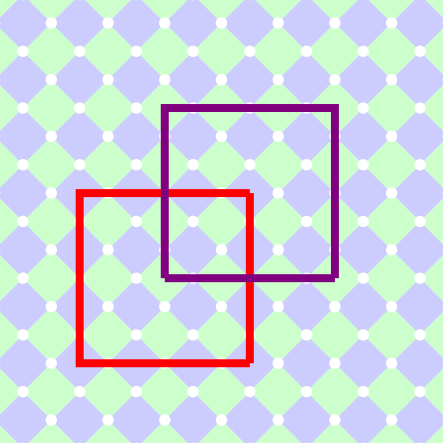

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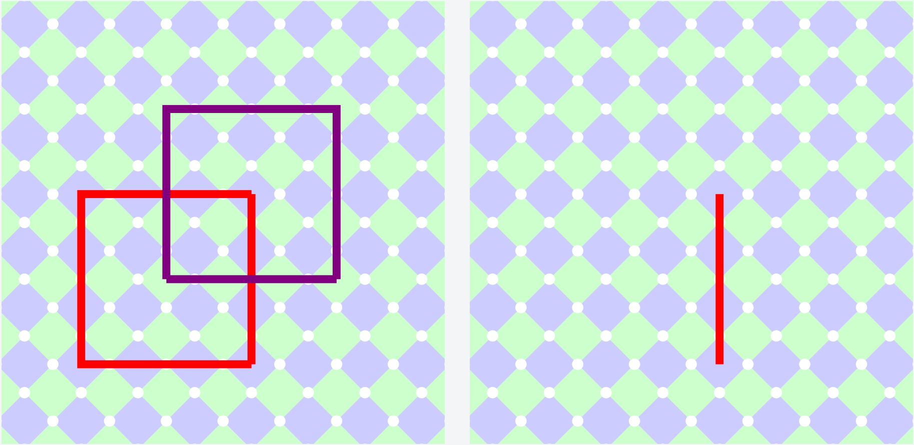

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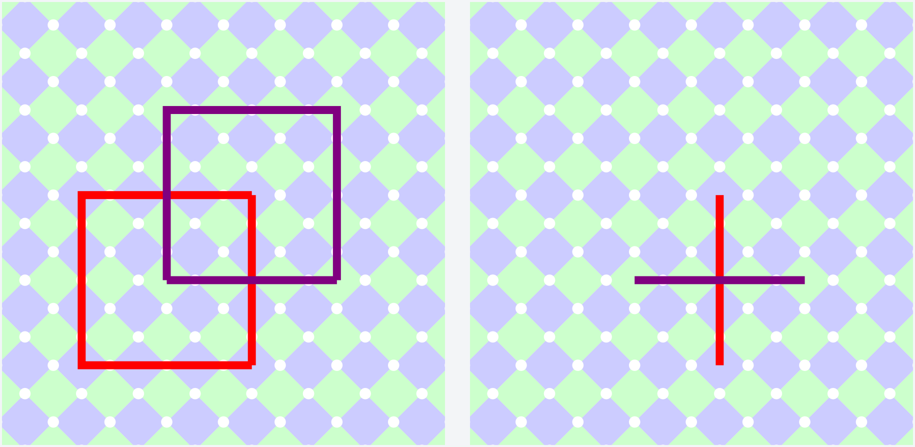

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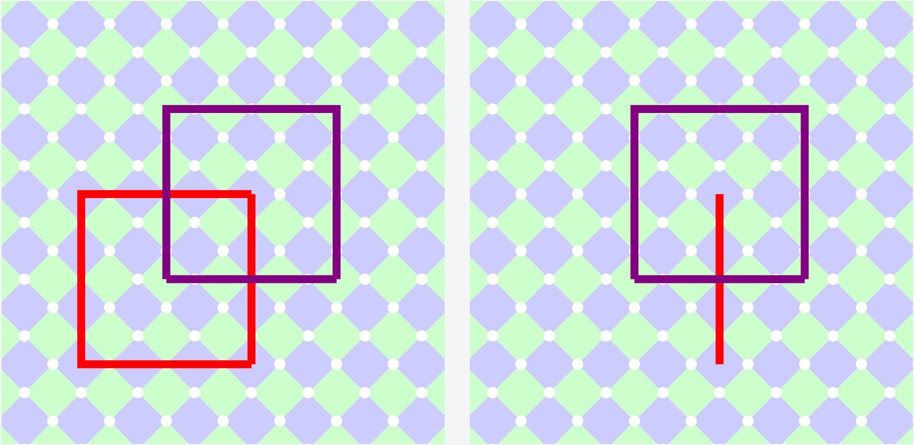

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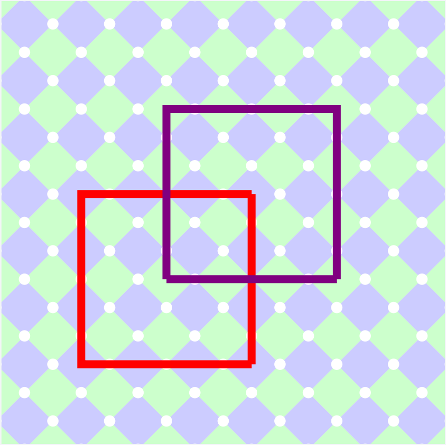

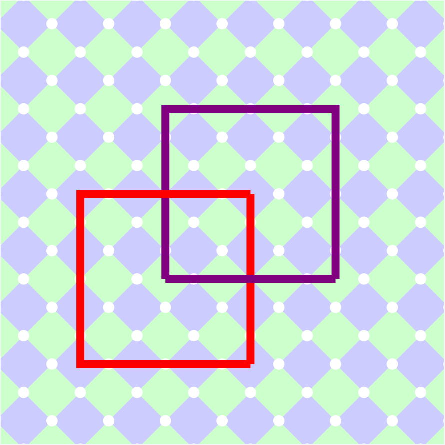

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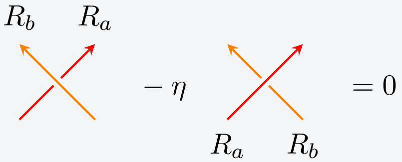

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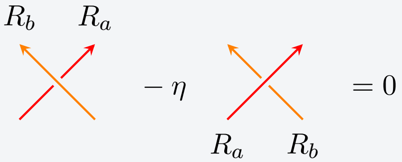

- Should not depend too much on the specific support chosen
  - Deformable

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- Algorithm
  - (Modified) DMRG

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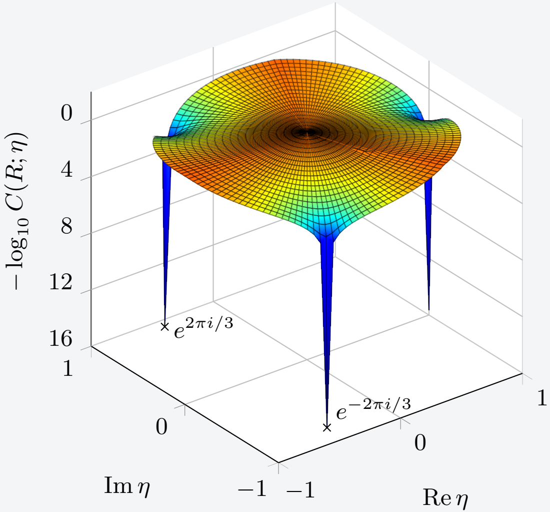

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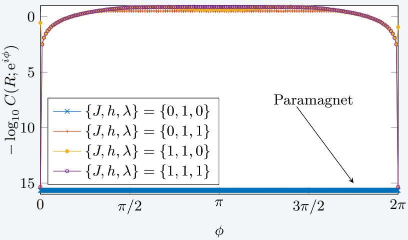
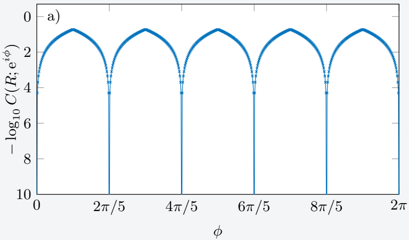
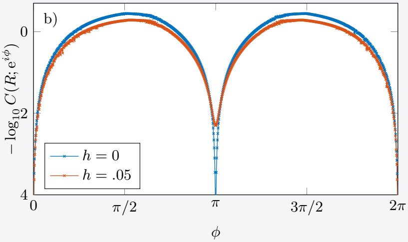
$$H = JH_{\mathbb{Z}_N \text{ Toric Code}} - \frac{h}{2}\sum(Z_j + Z_j^\dagger) - \frac{\lambda}{4}\sum_{\langle j,k\rangle}(X_j + X_j^\dagger)(X_k + X_k^\dagger)$$

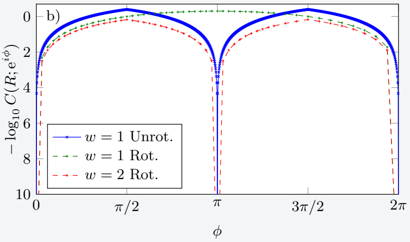




Toric Code with Z Field

$$H = H_{\mathbb{Z}_2 \text{ Toric Code}} - h \sum Z_j$$





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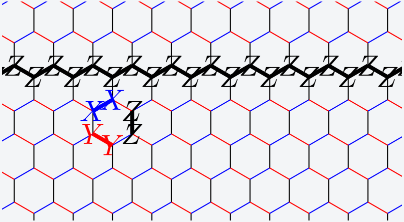

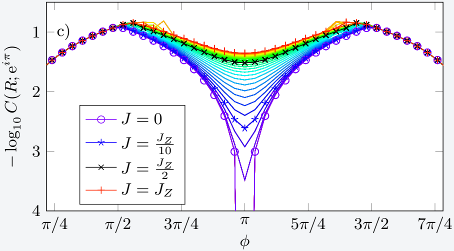

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- Can we prove anything about the method?
  - If we restrict to unitaries, we can prove ground state degeneracy using numerical output

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Topologically Encoded Qubit: Quantum Compass Model/Bacon Shor Code

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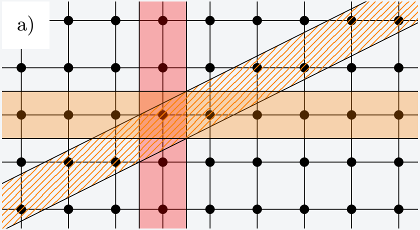

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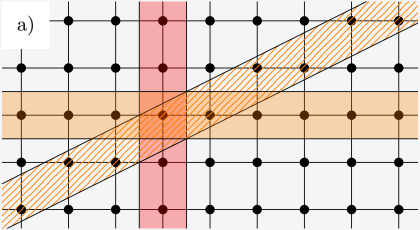

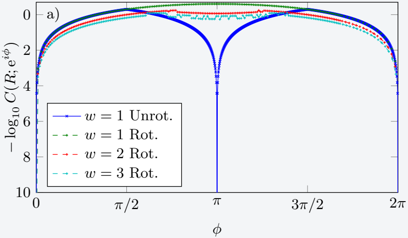

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