

Effective Edge States of SPT Systems

J.C. Bridgeman, J.C. Wang, L.H. Santos

Perimeter Scholars International

9th June 2014



perimeter scholars
international™

- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models



- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification



- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification
- Extract physical information about the model
 - Two methods recovering CFT data.



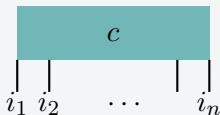
Tensor Network Formalism

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_n=1}^d c_{i_1 i_2 \dots i_n} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_n\rangle$$



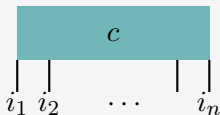
Tensor Network Formalism

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_n=1}^d c_{i_1 i_2 \dots i_n} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_n\rangle$$



Tensor Network Formalism

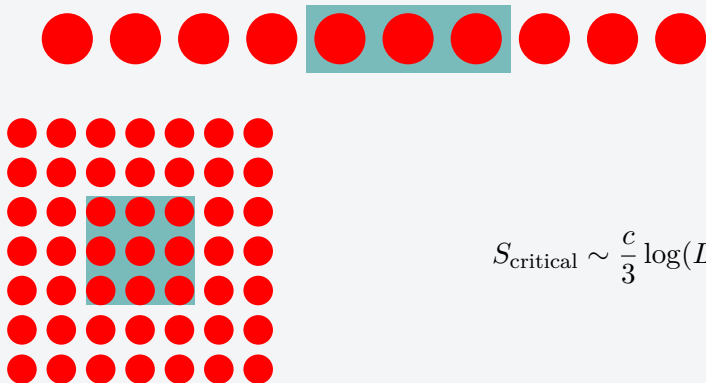
$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_n=1}^d c_{i_1 i_2 \dots i_n} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_n\rangle$$



$$A_a^b B_c^a = C_a^c \equiv \begin{array}{c} | \\ \boxed{B} \\ | \\ \boxed{A} \\ | \end{array} = \begin{array}{c} | \\ \boxed{C} \\ | \end{array}$$



Entanglement Entropy in Quantum States

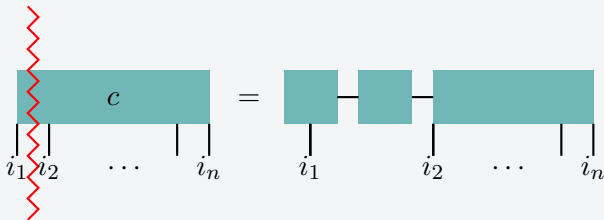


$$S_{\text{critical}} \sim \frac{c}{3} \log(L)$$



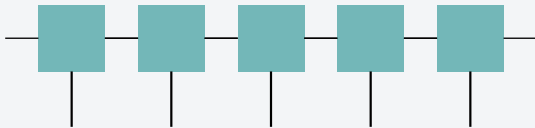
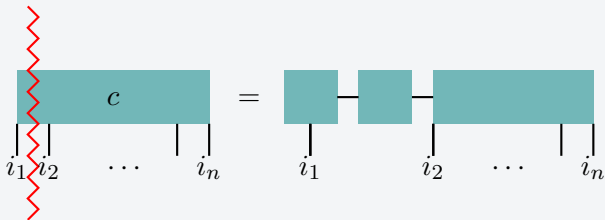
Matrix Product States

$$c_{(i_1)(i_2 i_3 \dots c_n)} = U_{i_1 j_1} S_{j_1 j_2} V_{(j_2)(i_2 i_3 \dots c_n)}$$



Matrix Product States

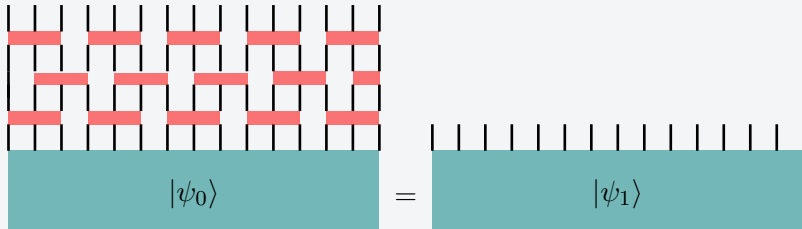
$$c_{(i_1)(i_2 i_3 \dots c_n)} = U_{i_1 j_1} S_{j_1 j_2} V_{(j_2)(i_2 i_3 \dots c_n)}$$



Phase Transitions



Phase Transitions



- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models



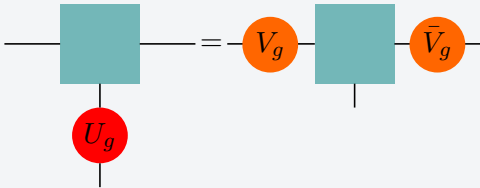
- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification



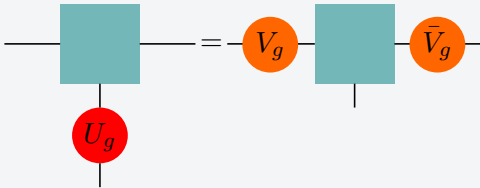
- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification
- Extract physical information about the model
 - Two methods recovering CFT data.



(1+1)-Dimensional SPT Classification



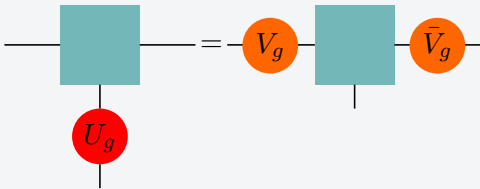
(1+1)-Dimensional SPT Classification



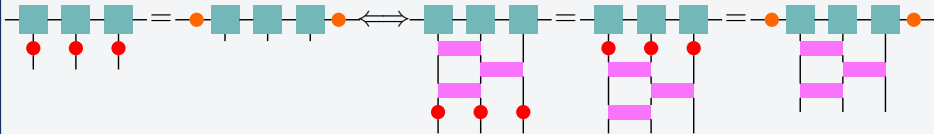
$$V_g V_h = \omega[g, h] V_{gh}$$



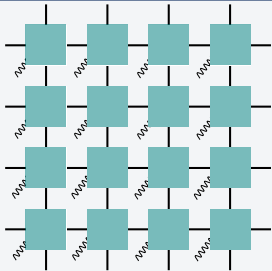
(1+1)-Dimensional SPT Classification



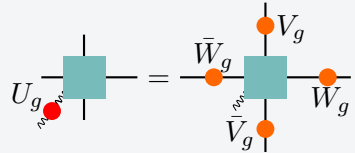
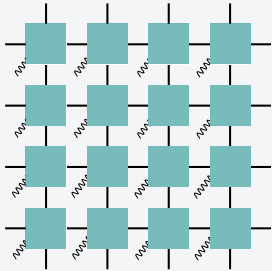
$$V_g V_h = \omega[g, h] V_{gh}$$



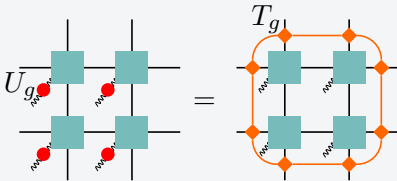
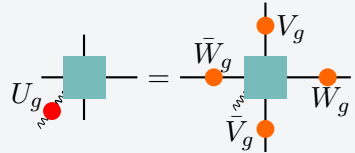
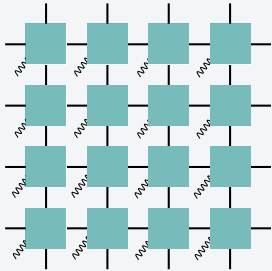
(2+1)-Dimensional SPT Classification?



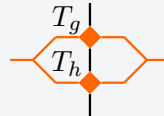
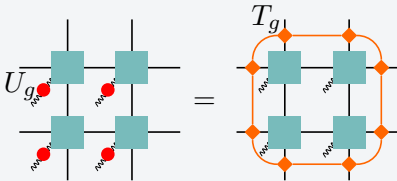
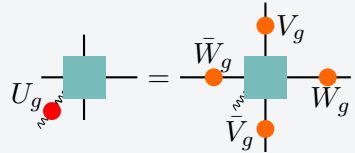
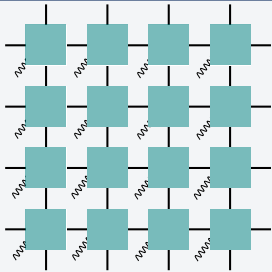
(2+1)-Dimensional SPT Classification?



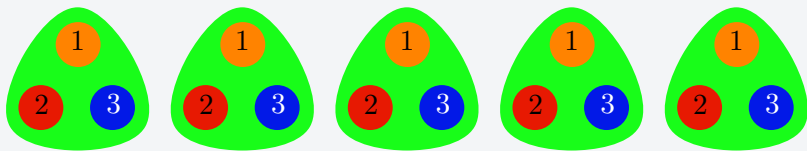
(2+1)-Dimensional SPT Classification?



(2+1)-Dimensional SPT Classification?



Constructing Models- $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ Symmetry



$$H = - \sum_{a=0}^1 \sum_{j=1}^M S^{-a} \left(X_j^{\{1\}} + X_j^{\{2\}} + X_j^{\{3\}} \right) S^a$$

L.H. Santos & J. Wang, Phys. Rev. B 89, 195122 (2014), arXiv:1310.8291.

J.C. Wang, L. Santos & X.G. Wen, arXiv:1403.5256



PERIMETER SCHOLARS
INTERNATIONAL

Constructing Models- $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ Symmetry



$$H = - \sum_{j=1}^M (2X_j^{\{1\}} + X_j^{\{2\}} + X_j^{\{3\}} + Z_{j-1}^{\{3\}} X_j^{\{2\}} Z_j^{\{3\}} + Z_j^{\{2\}} X_j^{\{3\}} Z_{j+1}^{\{2\}})$$



- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models



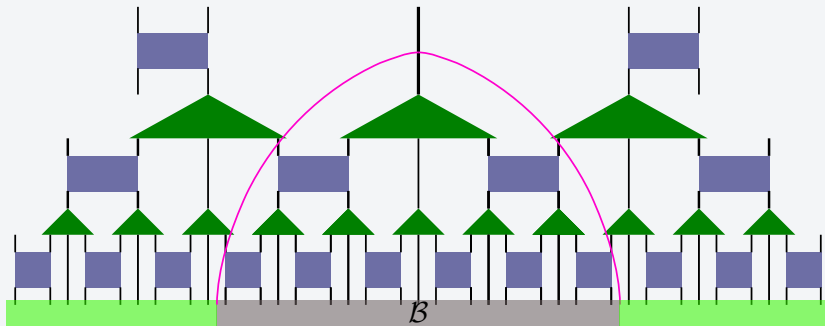
- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification

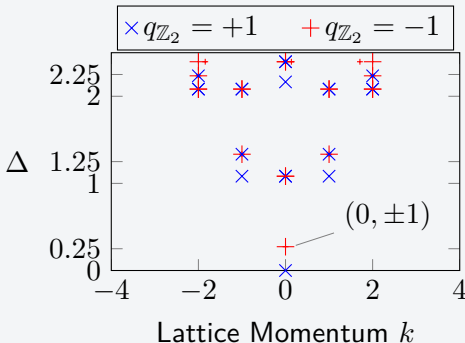
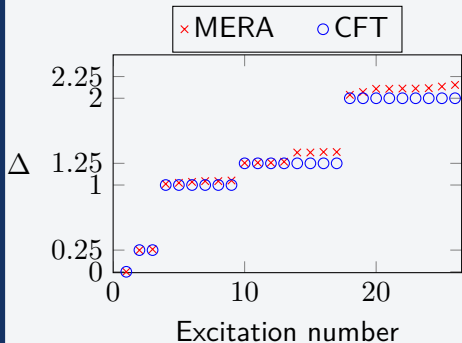


- Understanding quantum matter
 - (2+1)-dimensional symmetry protected topological models
- Non-onsite symmetry protecting gapless edge states
 - Ansatz based on group cohomology classification
- Extract physical information about the model
 - Two methods recovering CFT data.



Multiscale Entanglement Renormalisation Ansatz





States labelled by (m, n)

$$c_{\text{MERA}} = 1.0093$$

$$\Delta = \frac{n^2}{4} + m^2$$

$$q_{\mathbb{Z}_2} = \exp(i\pi(m+n))$$



- Motivated group cohomology classification for SPT phases



- Motivated group cohomology classification for SPT phases
- Non-onsite symmetry in $(2+1)$ dimensions



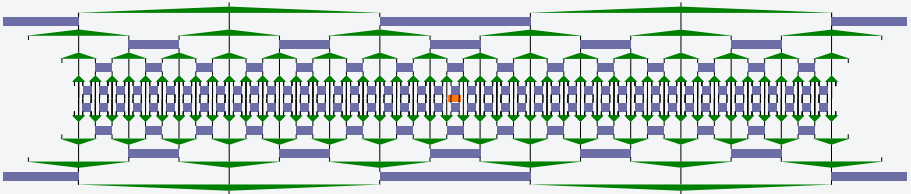
- Motivated group cohomology classification for SPT phases
- Non-onsite symmetry in $(2+1)$ dimensions
- Constructed a model for $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry



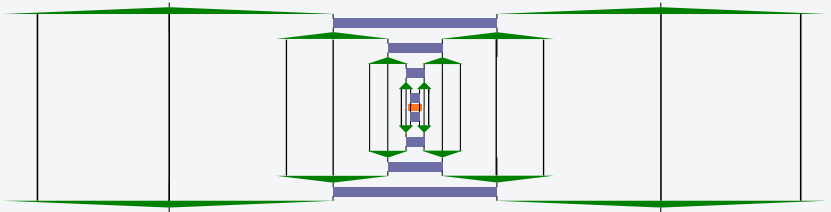
- Motivated group cohomology classification for SPT phases
- Non-onsite symmetry in $(2+1)$ dimensions
- Constructed a model for $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry
- CFT data consistent with $R = 2$ compactified free boson



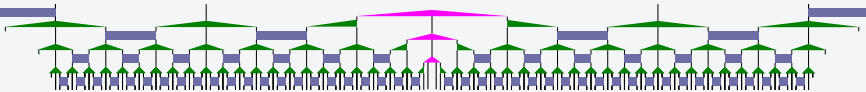
Impurity MERA



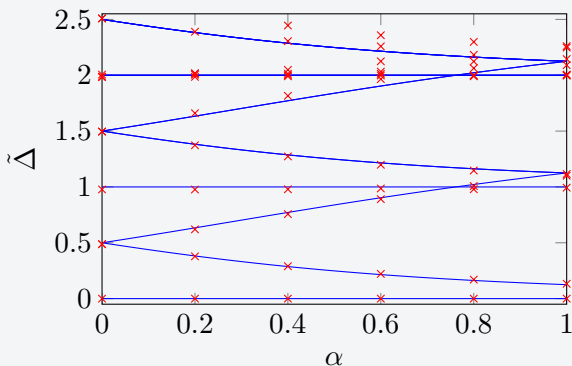
Impurity MERA



Impurity MERA



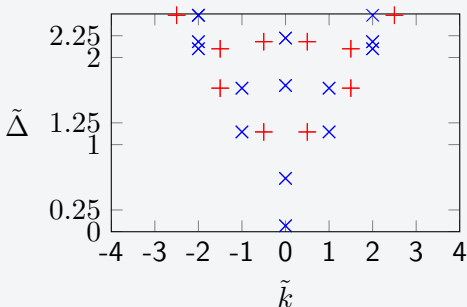
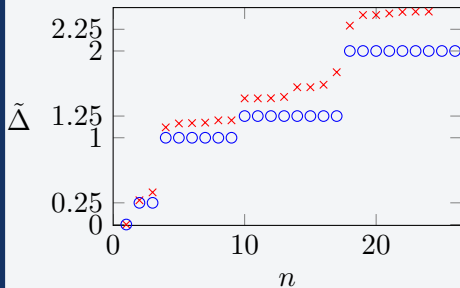
Benchmark Result-Ising with Defect



$$H = - \sum_{j=1}^M X_j - \sum_{j=1}^{M-1} Z_j Z_{j+1} - \alpha Z_M Z_1$$



Twisted Model



$$\tilde{\Delta} = \frac{(n + 1/2)^2}{4} + m^2$$

