Defects in topological phases

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Topological phase = (2+1)D string-net model

Defect = things we can add to the bare model

Boundaries, excitations, domain walls, . . .

Several ways of describing/ defining LW models

String types

Excitations

Lattice Hamiltonian

$$\text{String types} = \mathcal{C} \longrightarrow Z(\mathcal{C}) = \text{Excitations}$$

Fusion category

Modular category

Drawing silly pictures

$$|gs\rangle := |\quad\rangle + \alpha|\quad\rangle + \beta|\quad\rangle + \gamma|\quad\rangle + \dots$$

Declare this is a ground state.

Topological : Pictures related by "local moves" $\Rightarrow$ occur in superposition

Consistency $\Rightarrow$ Fusion category.

Brief intro to categories*

$$\mathcal{C}$$

set of objects

$$\{A, B, C, \dots\}$$

For each pair $A, B \in \mathcal{C}$

a set of morphisms $\mathrm{Hom}(A,B)$

$$f : A \longrightarrow B$$

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g \circ f"'] & B \arrow[d, "g"] \\
& C
\end{tikzcd}$$

* Skipping <u>many</u> important details

Example

Any set S as a category:

$|S|$ objects $\quad \mathrm{ob}\,\mathcal{C} = S$

$$\mathrm{Hom}(s, s') = \begin{cases} \text{identity} & \text{if } s = s' \\ \emptyset & s \neq s' \end{cases}$$

Example

Finite group G as a category:

1 object : $*$

$\mathrm{Hom}(*,*) = G$

$$\begin{tikzcd} * \arrow[r, "g"] \arrow[dr, "hg"'] & * \arrow[d, "h"] \\ & * \end{tikzcd}$$

Example : Vec : Category of $\mathbb{C}$-vector spaces

objects = {finite dimensional vector spaces}

$\mathrm{Hom}(V, W)$ = {linear maps $V \longrightarrow W$}.

simple objects : 1D vector space $\mathbb{C}$ (up to isomorphism)

All other objects can be written as

$$\mathbb{C} \oplus \mathbb{C} \oplus \ldots \oplus \mathbb{C}$$

Example $\mathrm{Vec}(G)$: G graded vector spaces

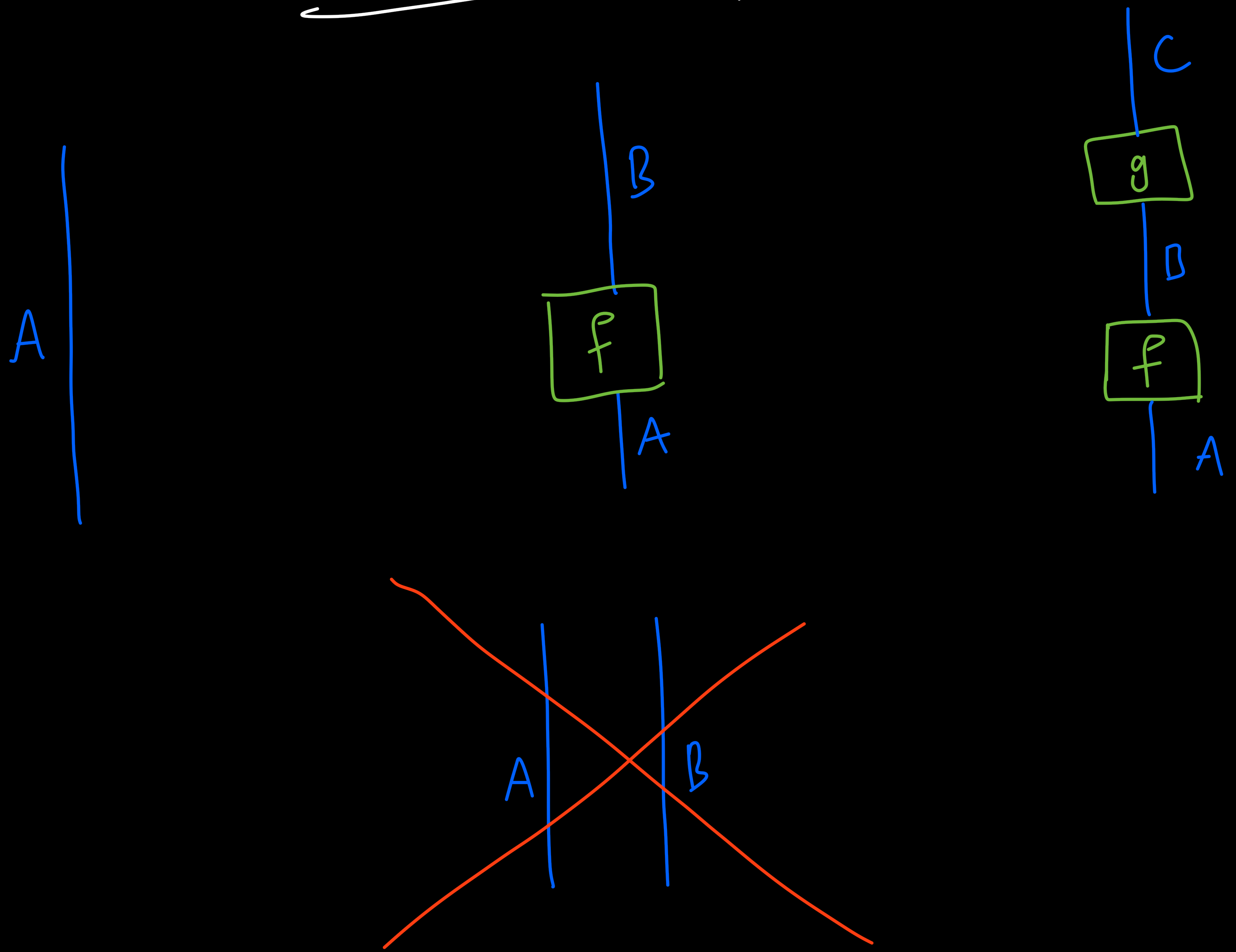
objects : $\{$ pairs $(V$ a vector space, $V = \bigoplus_g V_g)\}$.

simple objects : $(\mathbb{C}, g)$

morphisms : Linear maps $V \longrightarrow W$

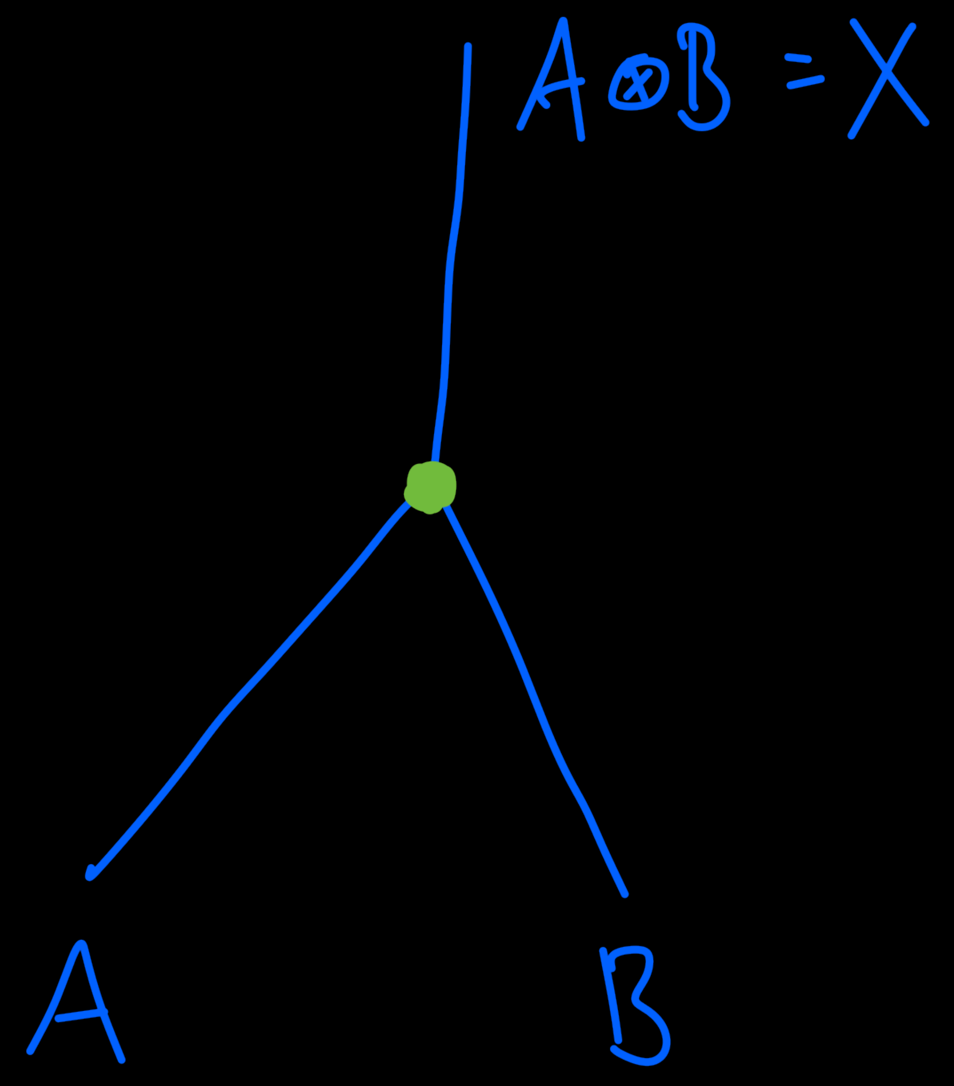
$f(V_g) \subset W_g$

Back to pictures

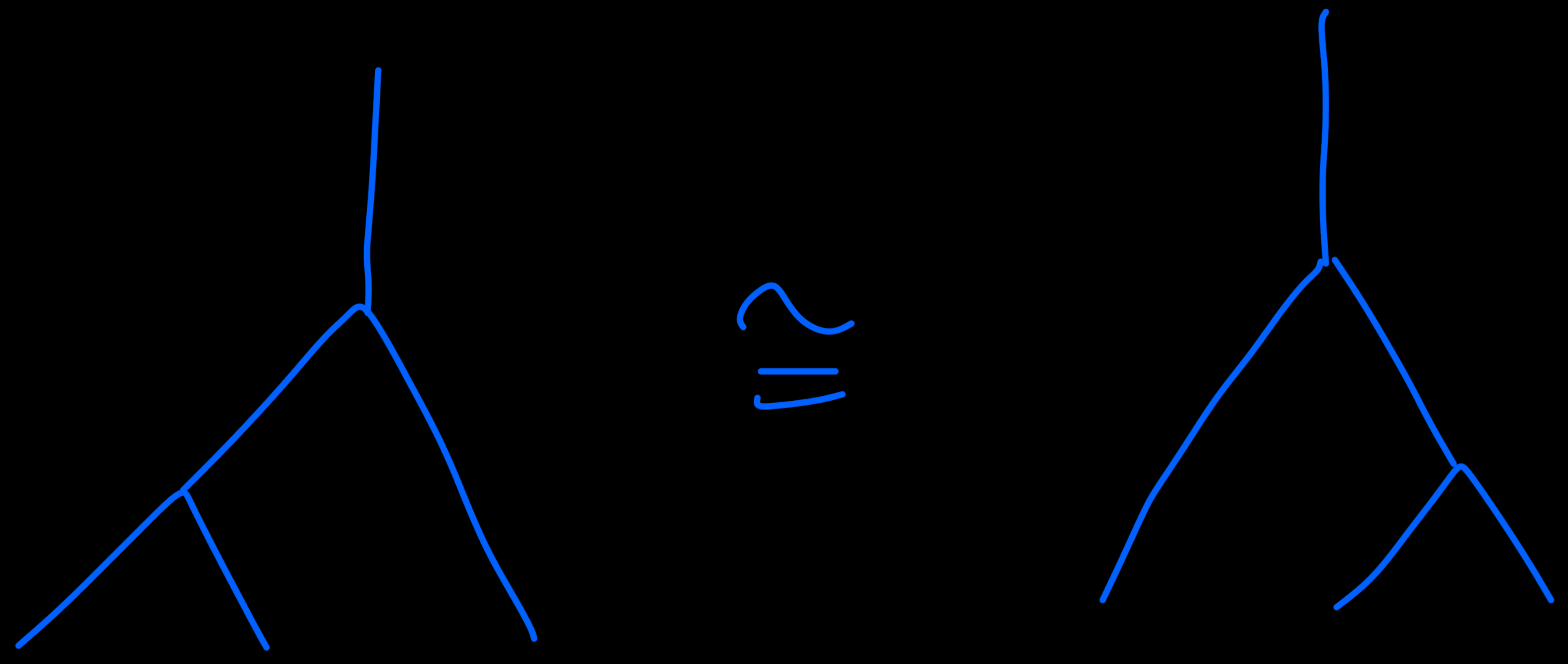


Fusion categories

$$\otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$$



$$(A \otimes B) \otimes C \cong A \otimes (B \otimes C)$$



Example

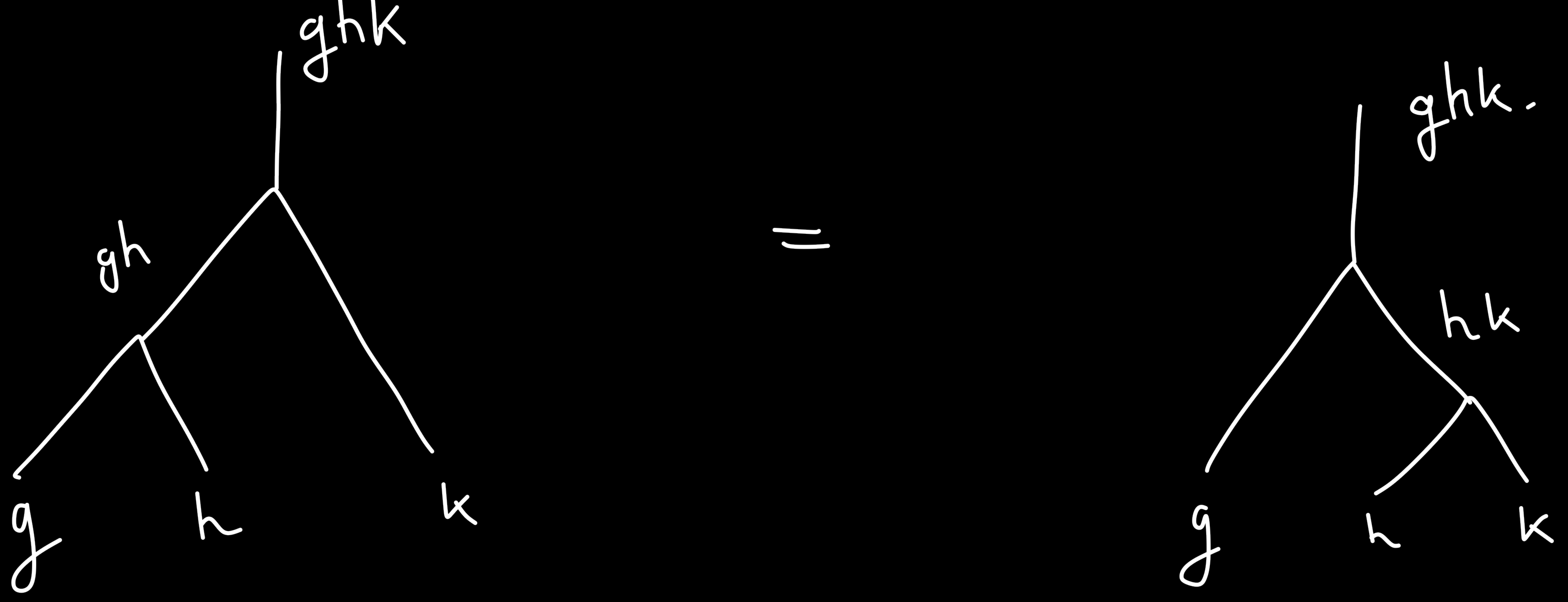
Vec as a FC:

$$\mathbb{C}^m \otimes \mathbb{C}^n \longrightarrow \mathbb{C}^{mn}$$

Example

$\mathrm{Vec}(G)$

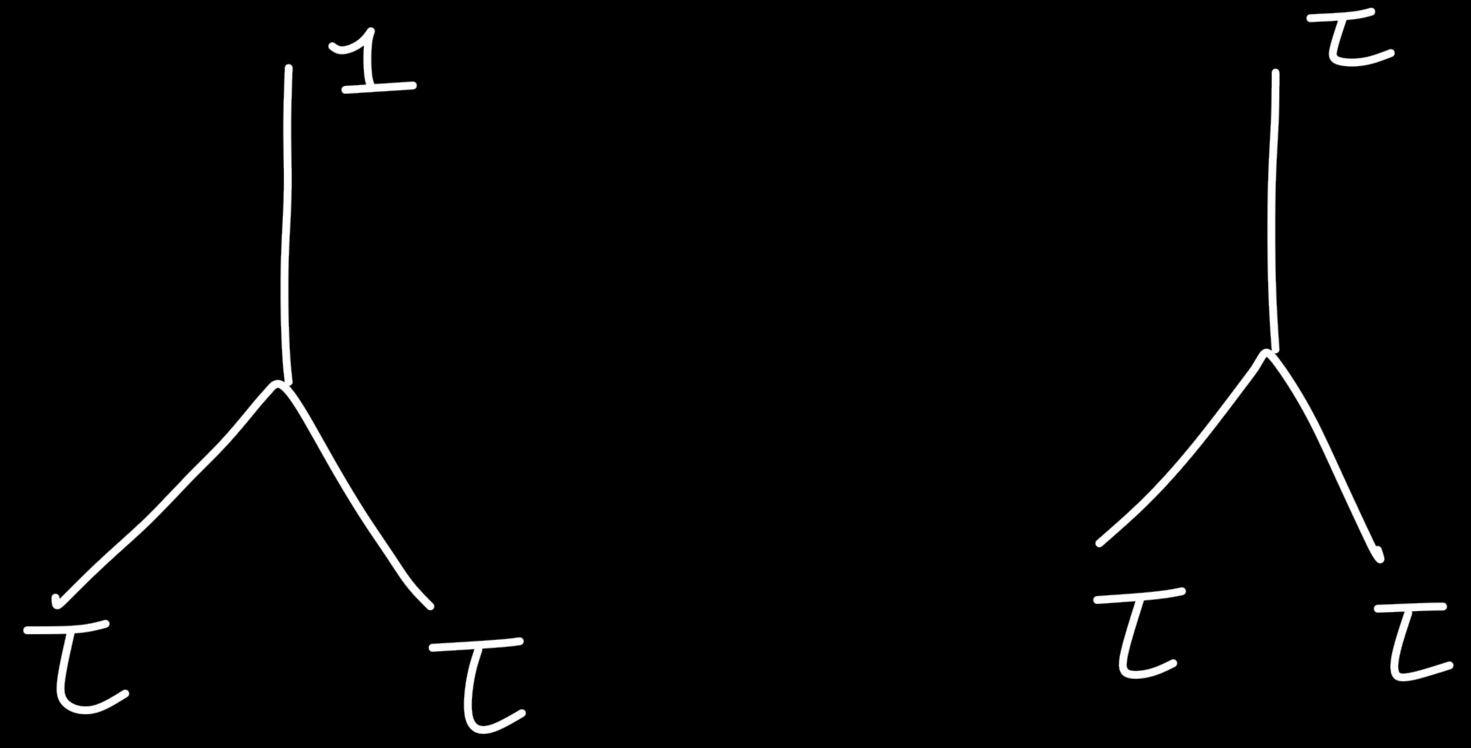
$$(\mathbb{C}, g) \otimes (\mathbb{C}, h) = (\mathbb{C}, gh)$$



Example

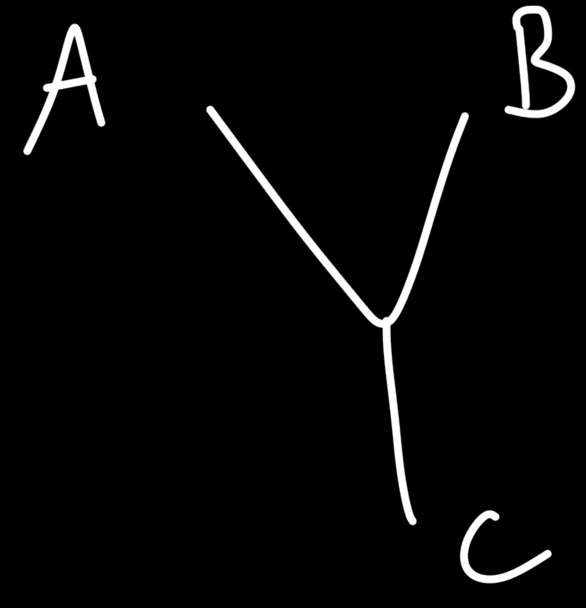
$\mathrm{Fib}:\quad \{1, \tau\}.$

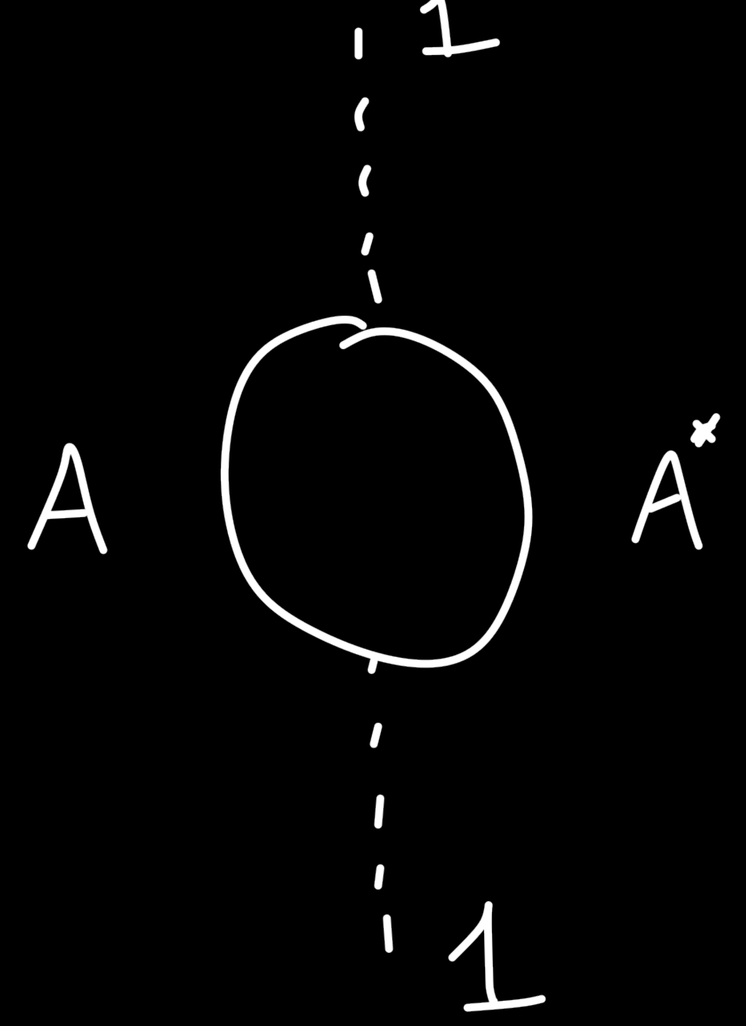
$$\tau \otimes \tau = 1 \oplus \tau$$



Need some notion of conjugation

$\Rightarrow$ flip pictures upside down





$$\in \mathrm{Hom}(1, 1) \cong \mathbb{C}$$

String-net models

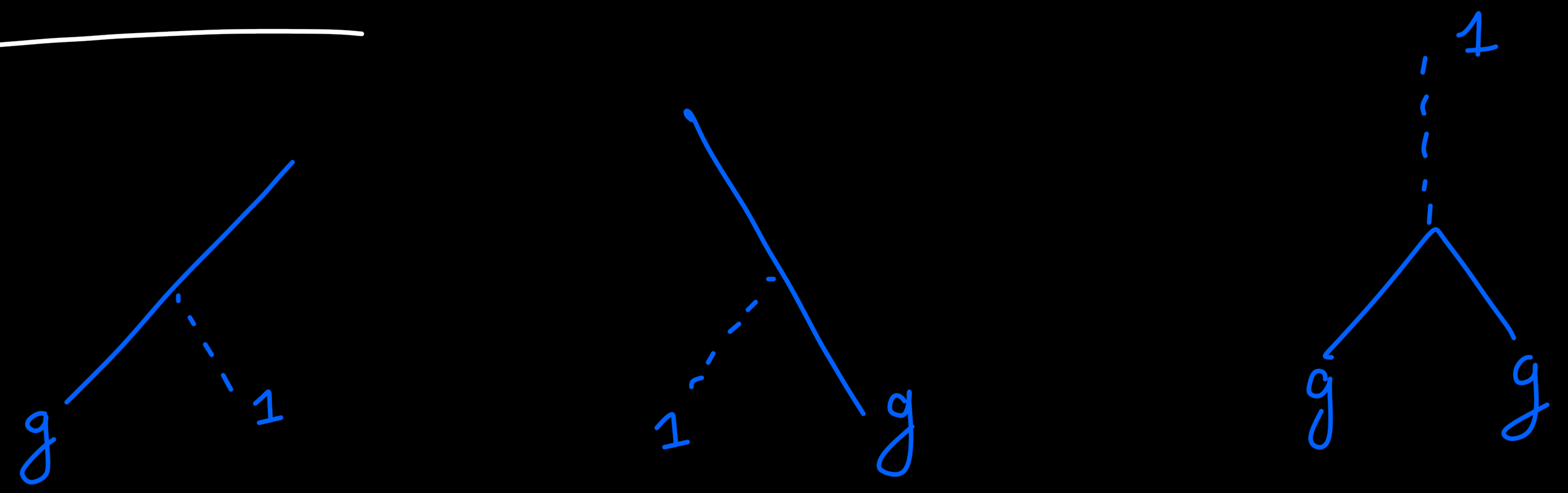
1) Draw a closed picture from $\mathcal{C}$.

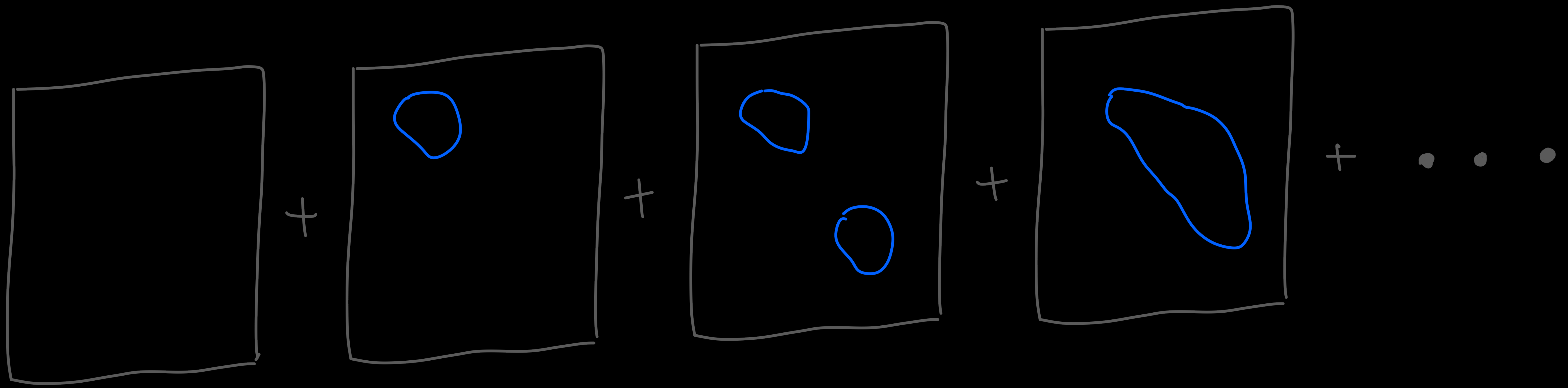
2) Keep adding pictures obtained by local moves

3) $\Longrightarrow$ ground state

Example

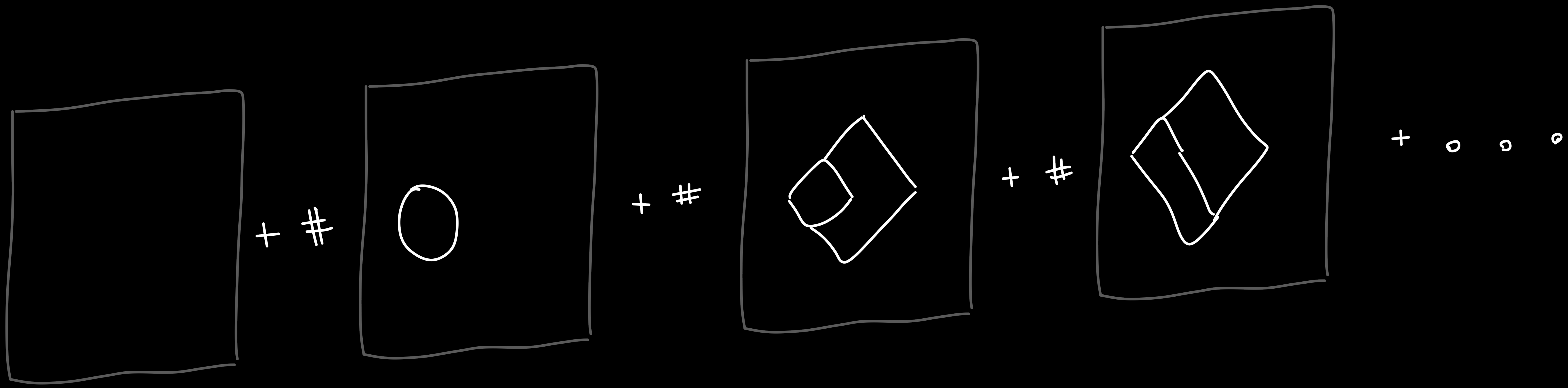
<u>Toric code</u> = $\mathrm{Vec}\,\mathbb{Z}/2 = \{1, g\}$.





Example

Double Fibonacci

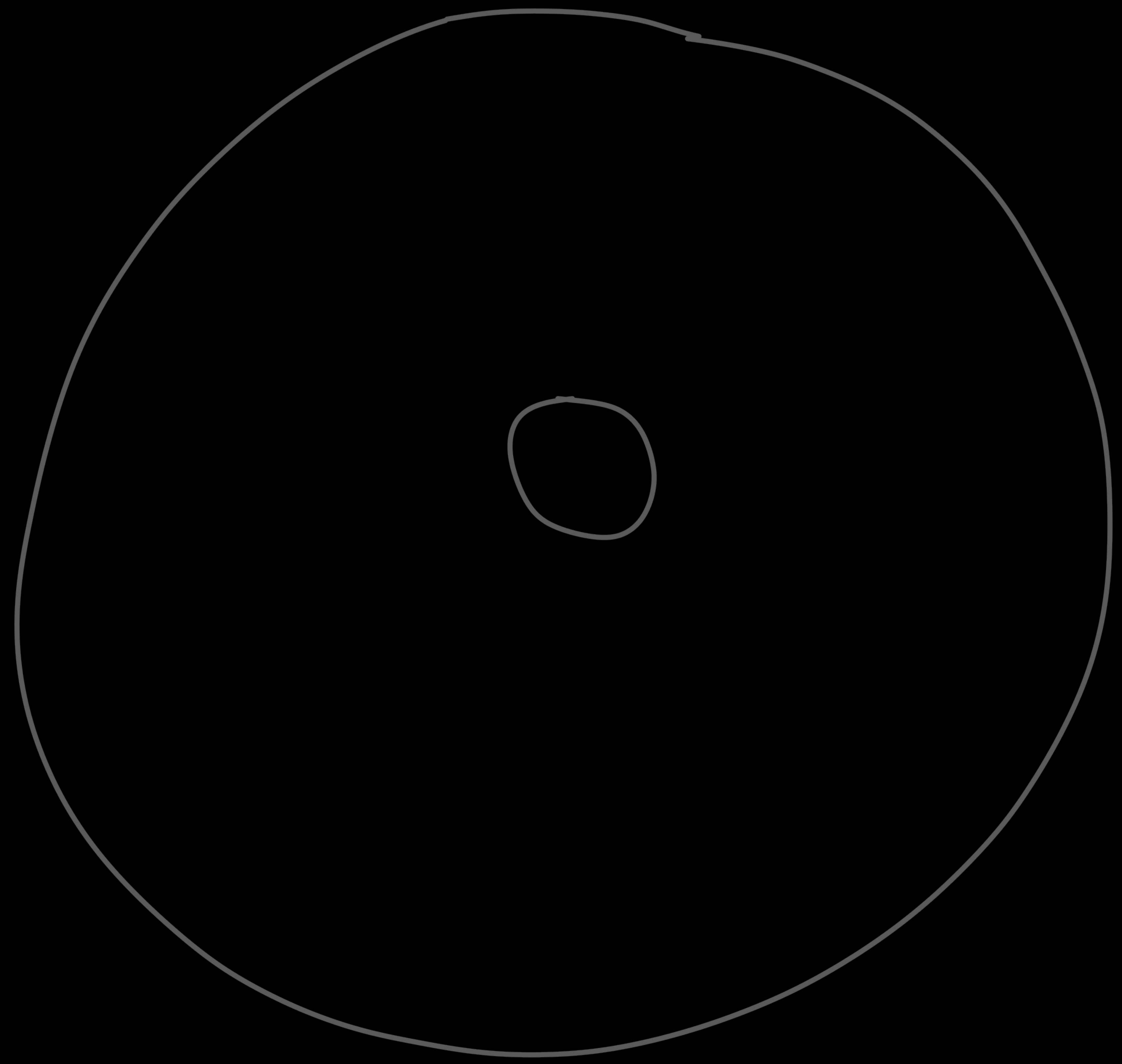


That's bare string nets

Questions ?

Excitations

Modify the pictures at a point



Excitations

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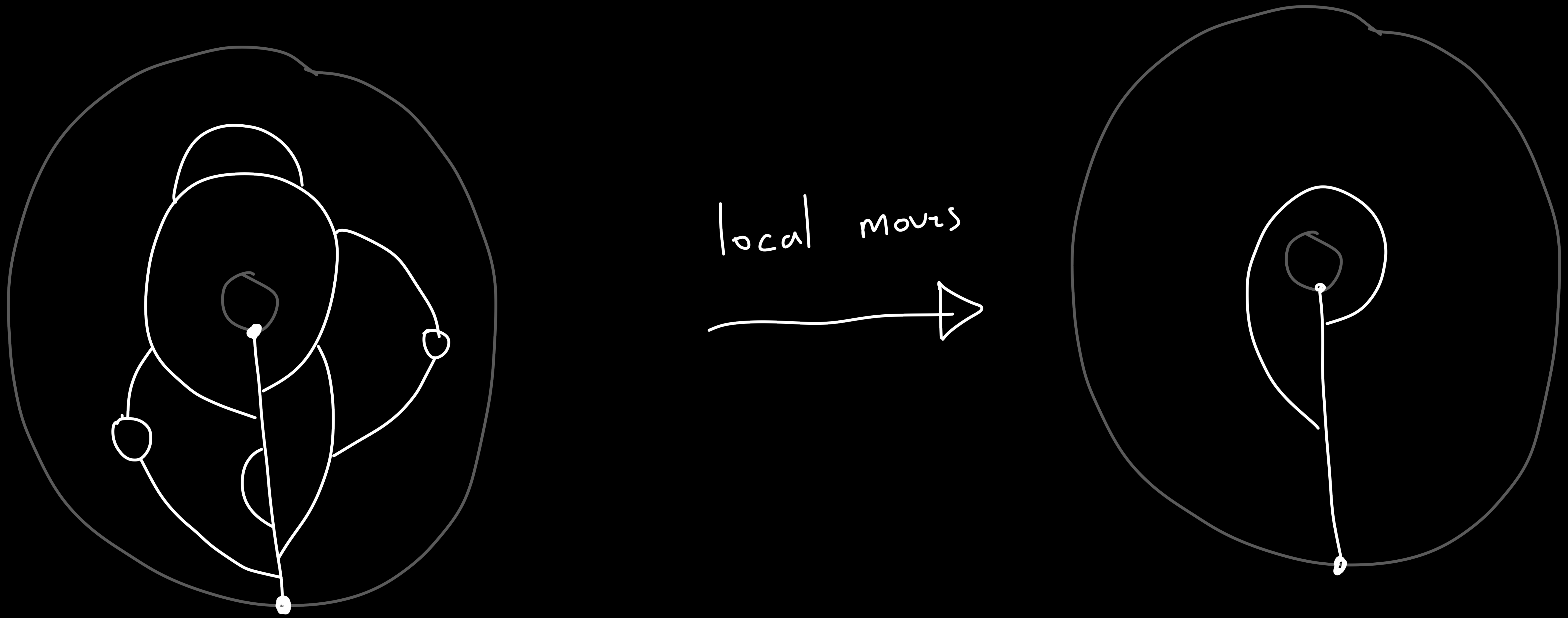
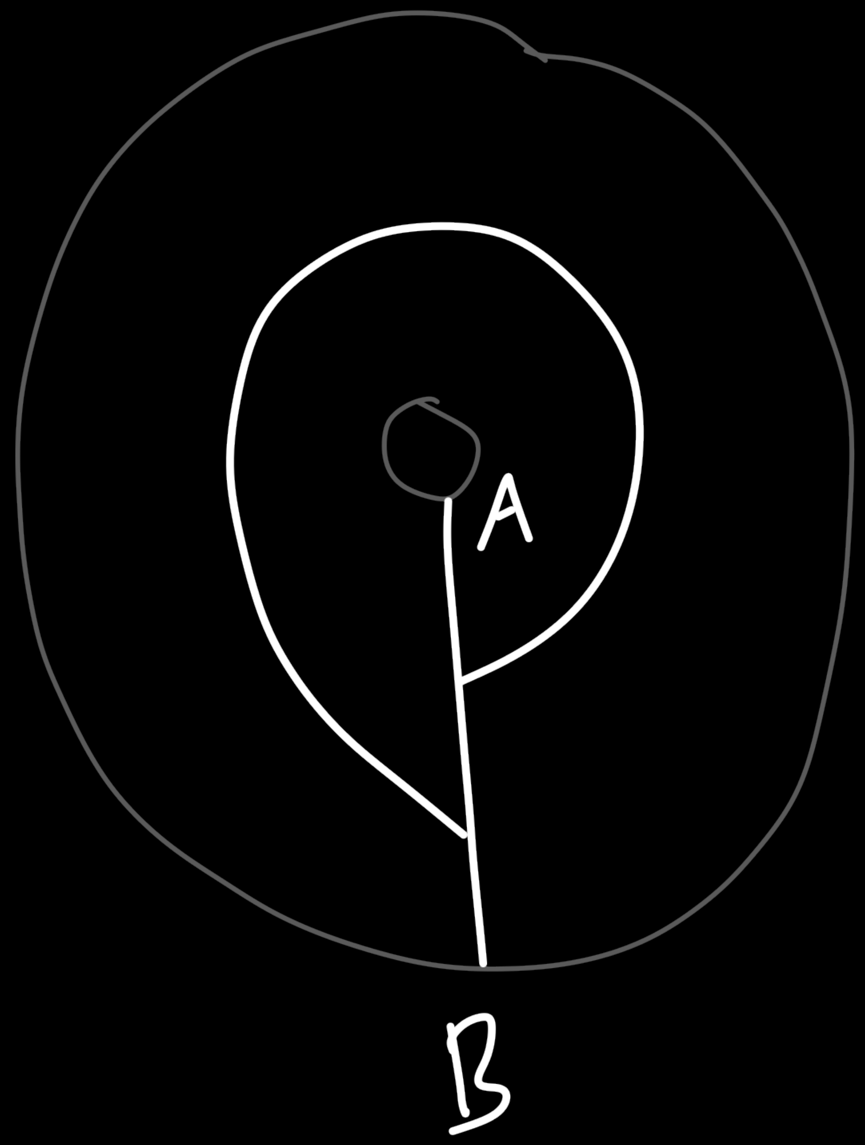


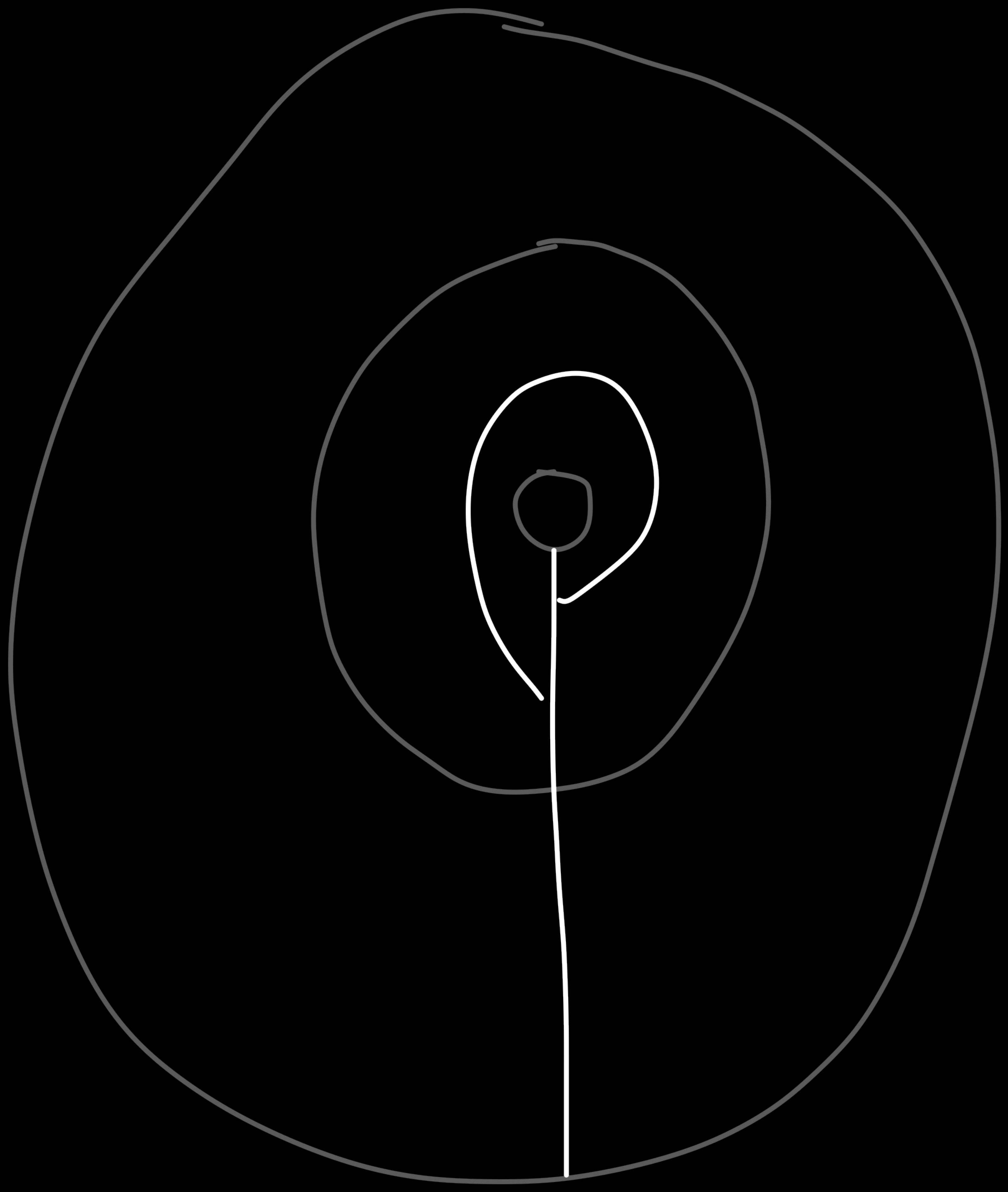
Tube category

morphisms



$: A \longrightarrow B$

composition



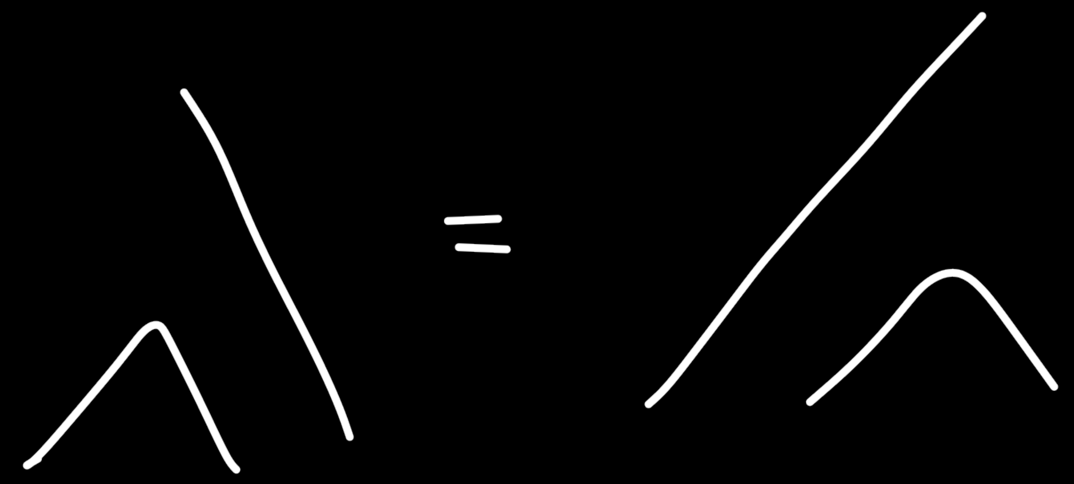
Excitations = Irreducible representations of tube category

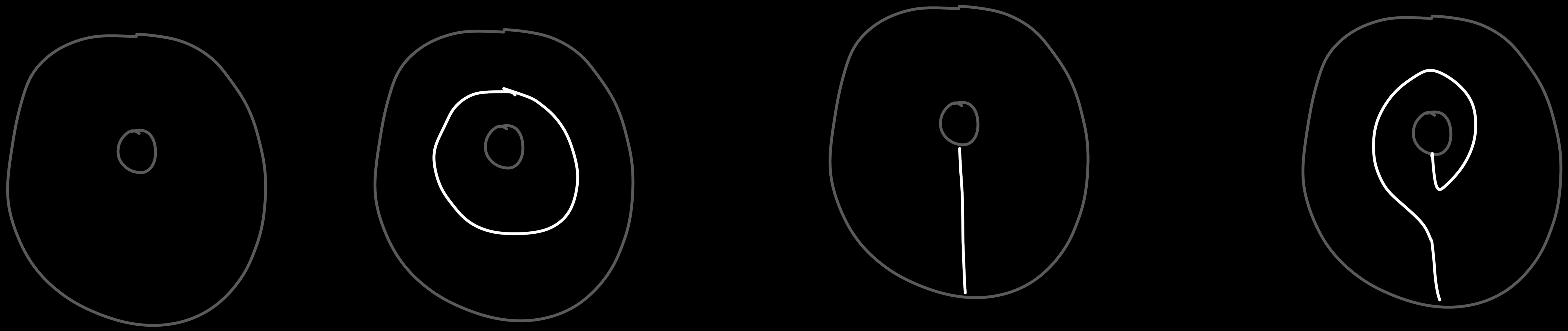
Categorical representation: object $A \longmapsto$ Vector space V_A

$(f : A \longrightarrow B) \longmapsto (L_f : V_A \longrightarrow V_B)$

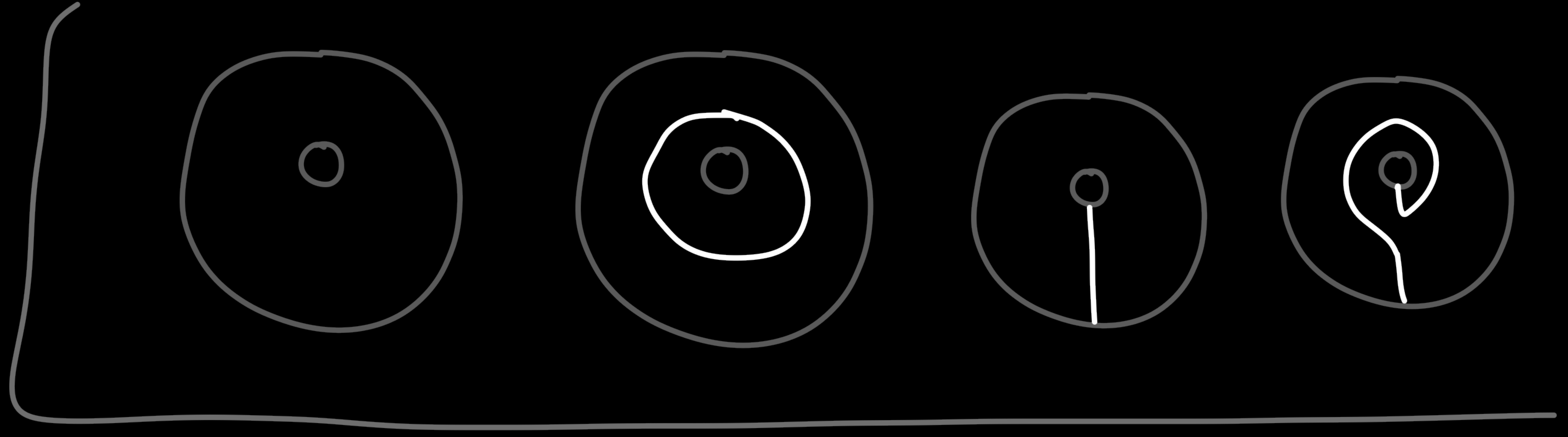
Example

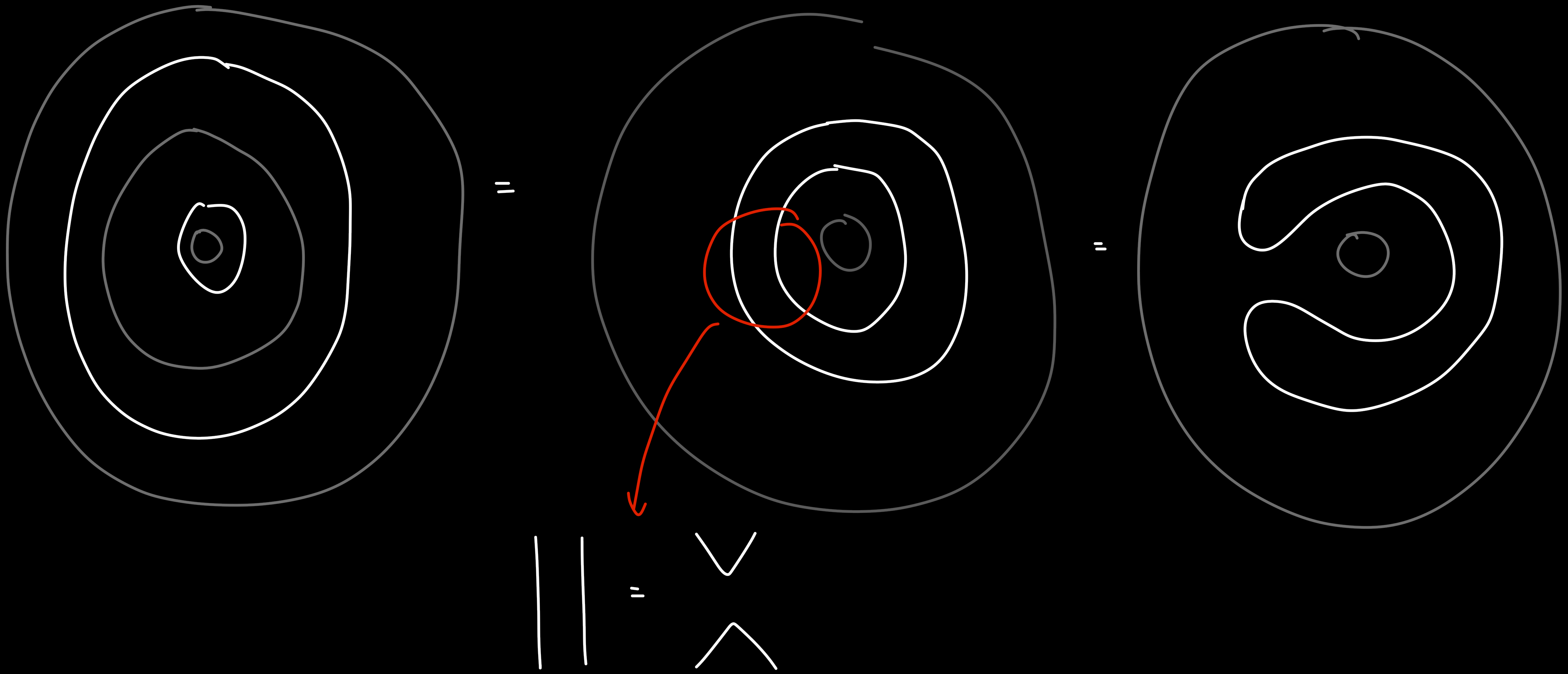
Toric code = $\mathrm{Vec}(\mathbb{Z}/2)$



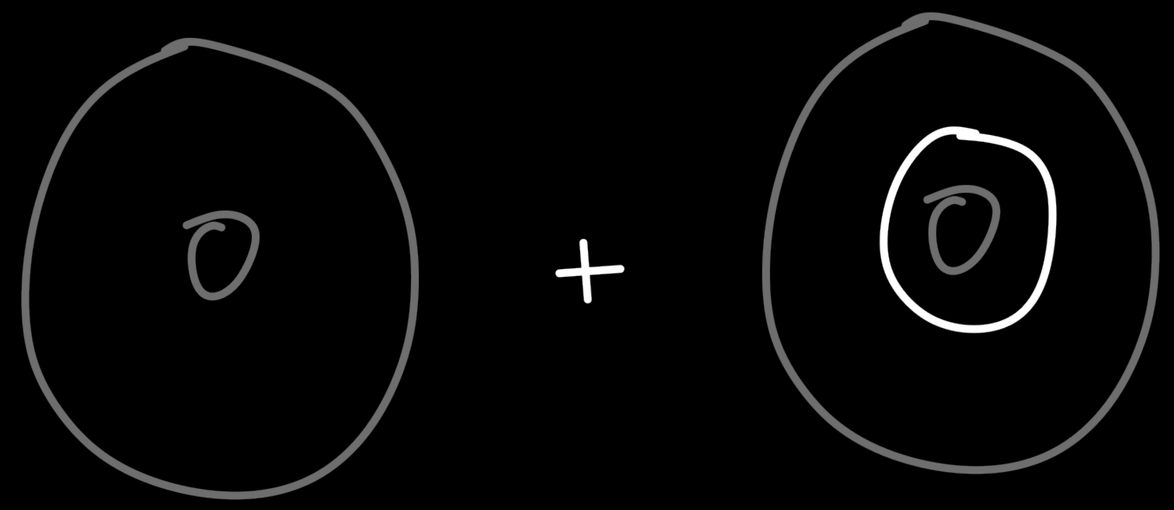


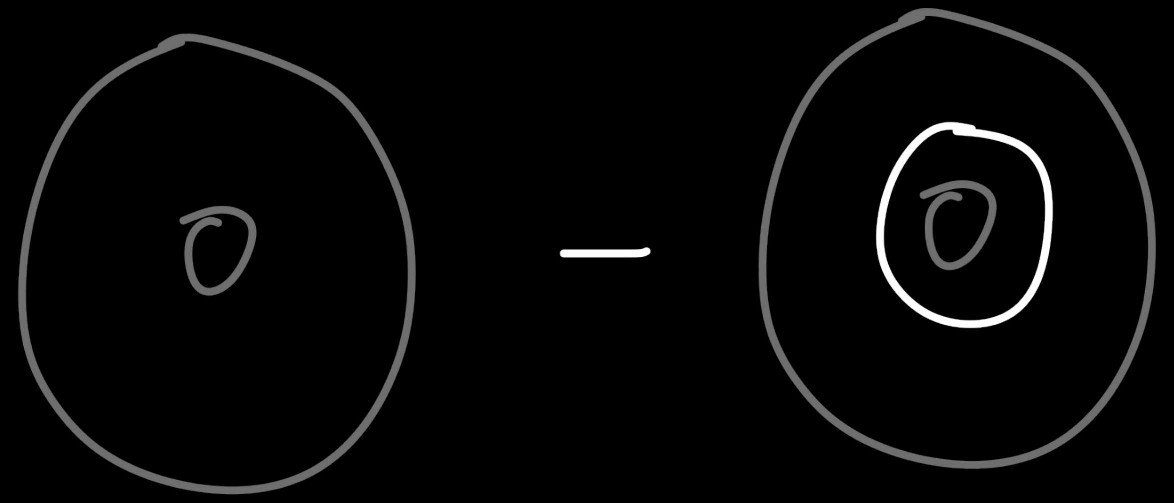
Example

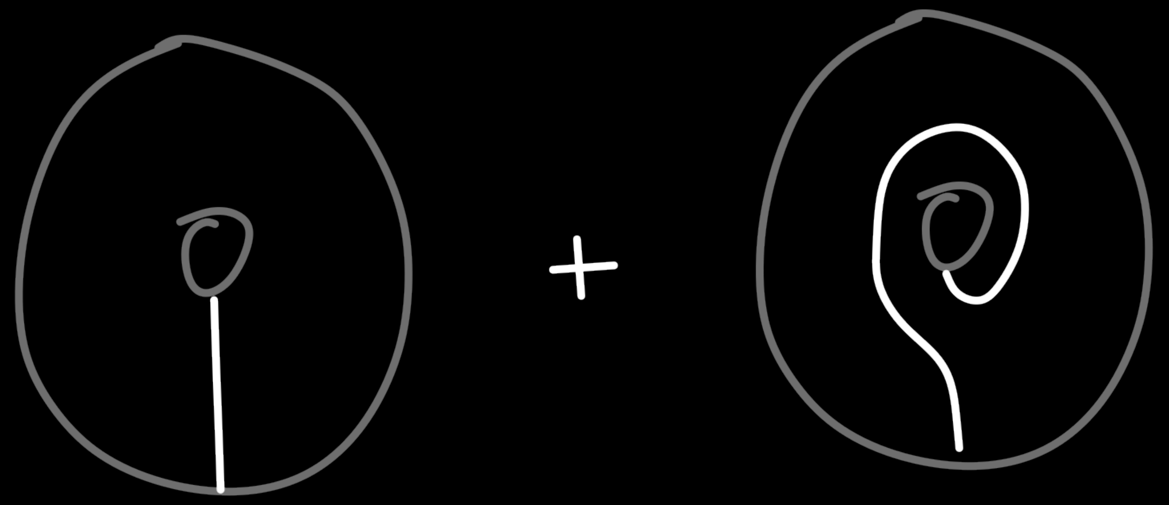


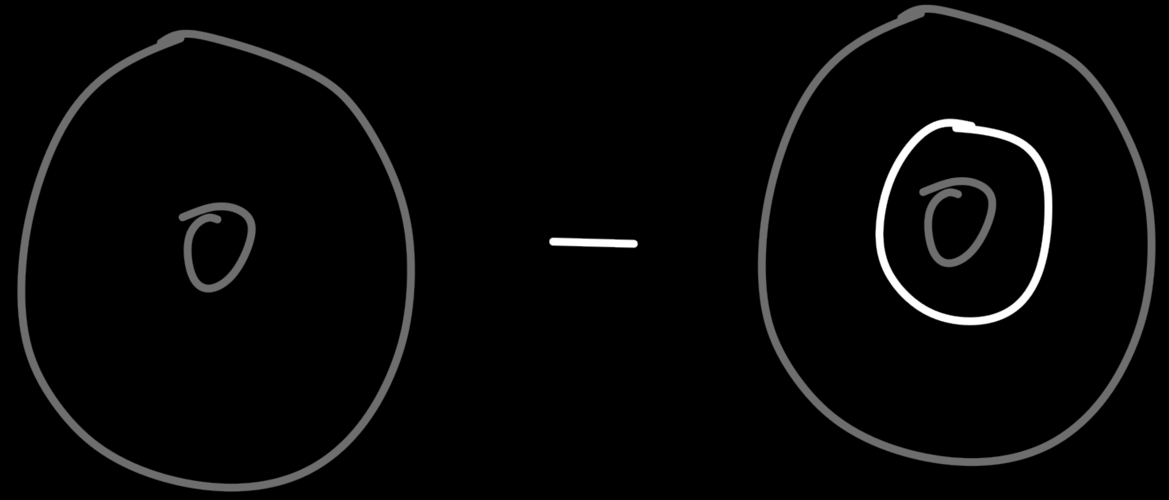


Example

$P_1 = \frac{1}{2}\Big($  $\Big)$

$P_e = \frac{1}{2}\Big($  $\Big)$

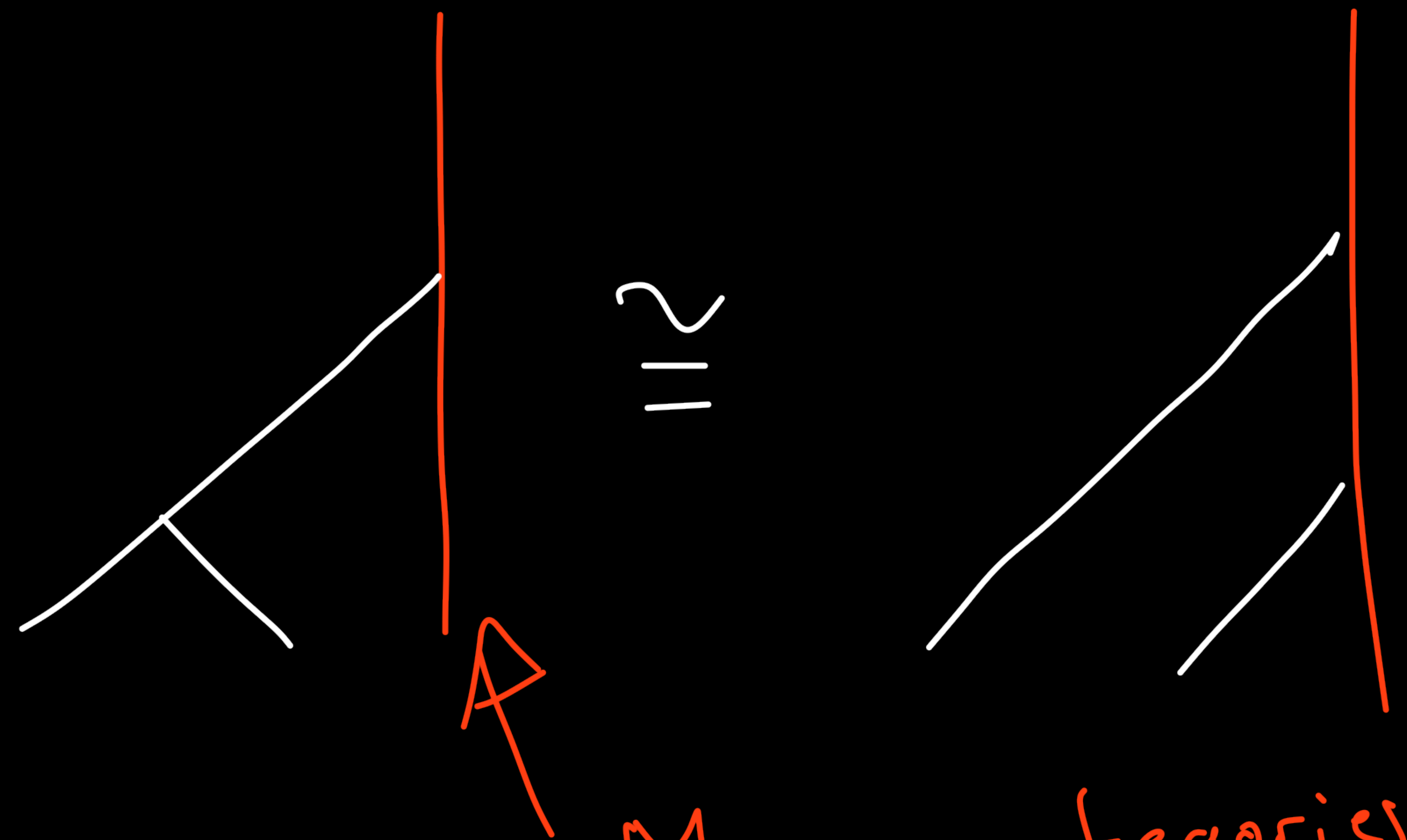
$P_m = \frac{1}{2}\Big($  $\Big)$

$P_{em} = \frac{1}{2}\Big($  $\Big)$

e
m
=
1/2
x
1/2
= 1/4
= 1/4
= − 1/2
= −
m
e

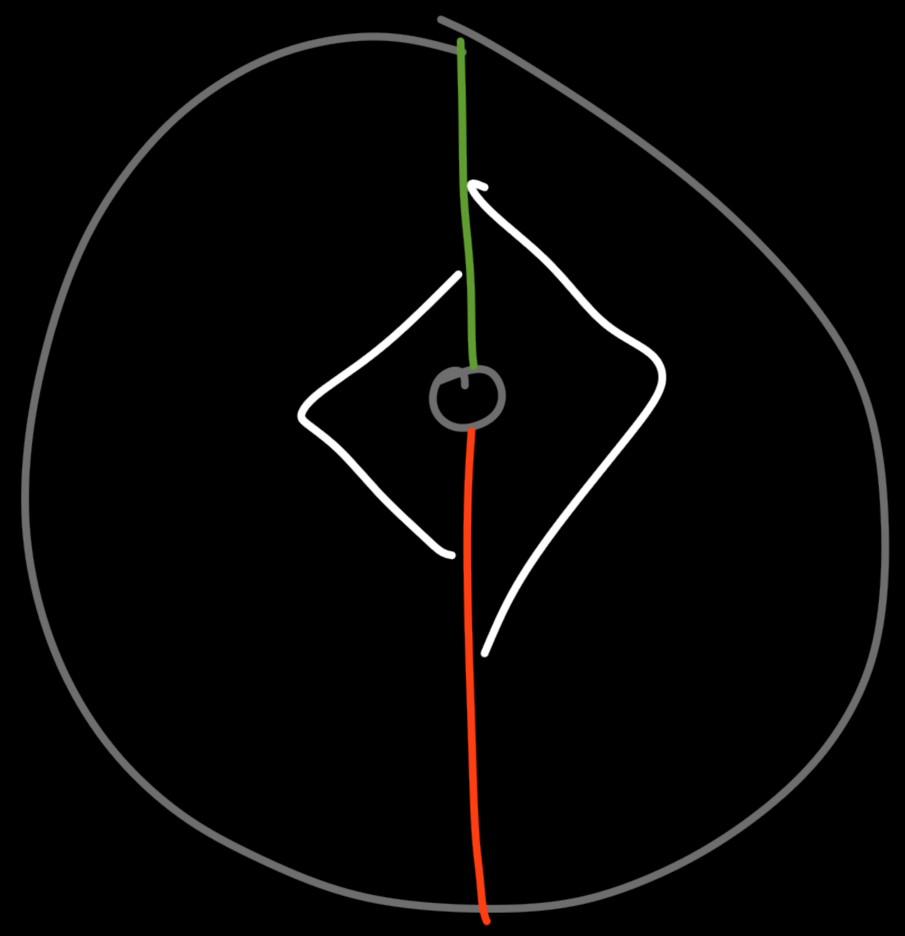
Can compute fusion in the same way

Boundaries : How can we modify a 1D region



More categories,
this one a "module category"

Boundary excitations :
combine these ideas



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Questions?

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"Tensor categories: Etingof et al."