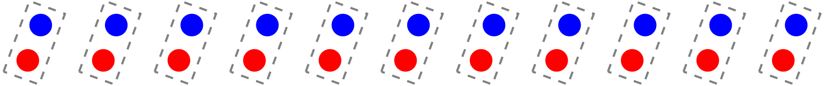
Anomalies and entanglement renormalization

Jacob Bridgeman and Dominic Williamson

arXiv:1703.07782

A Motivating Example

$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_j Z_{j+1} + \tilde{Z}_j\tilde{Z}_{j+1}) - c\sum(Z_j\tilde{X}_j Z_{j+1} + \tilde{Z}_j X_{j+1}\tilde{Z}_{j+1})$$



$$(1,0) \mapsto \bigotimes_j X_j, \qquad (0,1) \mapsto \bigotimes_j \tilde{X}_j$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ on-site symmetry

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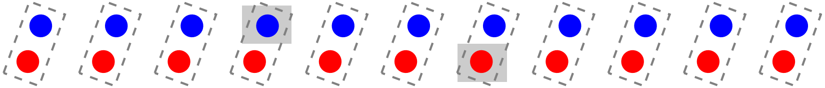

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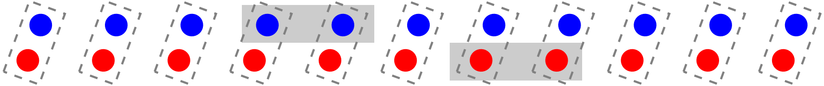

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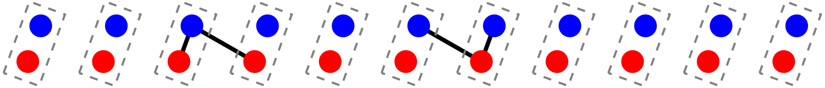

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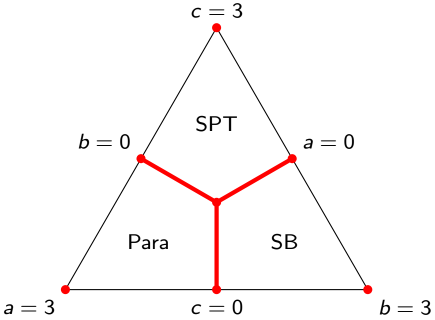
$\mathbb{Z}_2 \times \mathbb{Z}_2$ on-site symmetry

Phase Diagram

$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_j Z_{j+1} + \tilde{Z}_j \tilde{Z}_{j+1}) - c\sum(Z_j \tilde{X}_j Z_{j+1} + \tilde{Z}_j X_{j+1} \tilde{Z}_{j+1})$$



$$a + b + c = 3$$

Duality Transformations

$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_j Z_{j+1} + \tilde{Z}_j\tilde{Z}_{j+1}) - c\sum(Z_j\tilde{X}_j Z_{j+1} + \tilde{Z}_j X_{j+1}\tilde{Z}_{j+1})$$

- ISING

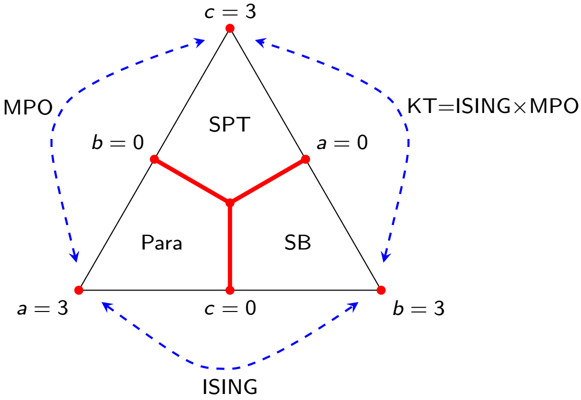
$$X_j \leftrightarrow Z_{j-1}Z_j$$
$$\tilde{X}_j \leftrightarrow \tilde{Z}_j\tilde{Z}_{j+1}$$
$$a \leftrightarrow b$$

- MPO

$$X_j \leftrightarrow \tilde{Z}_{j-1}X_j\tilde{Z}_j$$
$$\tilde{X}_j \leftrightarrow Z_j\tilde{X}_j Z_{j+1}$$
$$a \leftrightarrow c$$

Phase Diagram

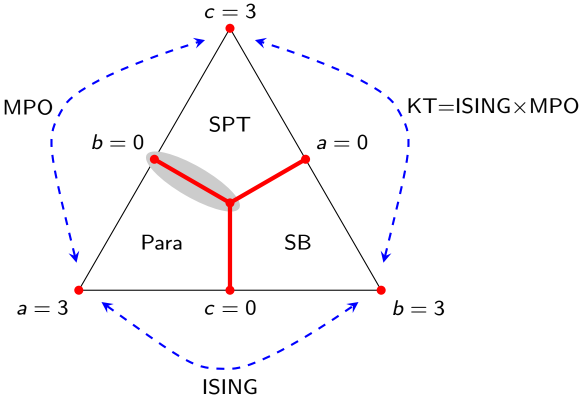
$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_jZ_{j+1} + \tilde{Z}_j\tilde{Z}_{j+1}) - c\sum(Z_j\tilde{X}_jZ_{j+1} + \tilde{Z}_jX_{j+1}\tilde{Z}_{j+1})$$



$$a + b + c = 3$$

Phase Diagram

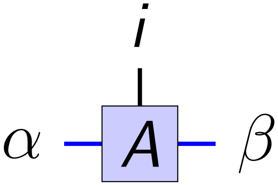
$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_jZ_{j+1} + \tilde{Z}_j\tilde{Z}_{j+1}) - a\sum(Z_j\tilde{X}_jZ_{j+1} + \tilde{Z}_jX_{j+1}\tilde{Z}_{j+1})$$



$$a + b + c = 3$$

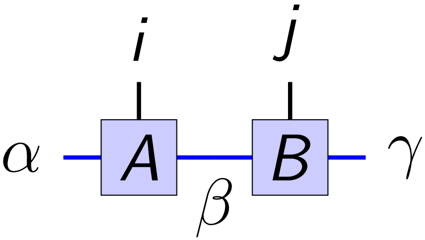
Tensor Networks

Tensor



$$= (A^{i})_{\alpha\beta} \in \mathbb{C}$$

Contraction



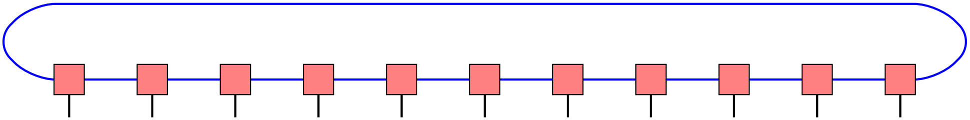
$$= \sum_{\beta} (A^i)_{\alpha\beta} (B^j)_{\beta\gamma}$$

$$= (A^i B^j)_{\alpha\gamma}$$

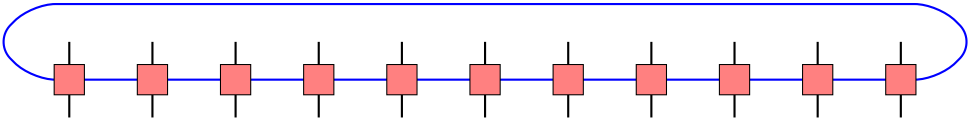
Matrix Product State

$$|\psi\rangle = \sum_{i_1, i_2, \cdots, i_N = 0}^{d-1} \mathrm{Tr}\left(A_{i_1} A_{i_2} \cdots A_{i_N}\right) |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

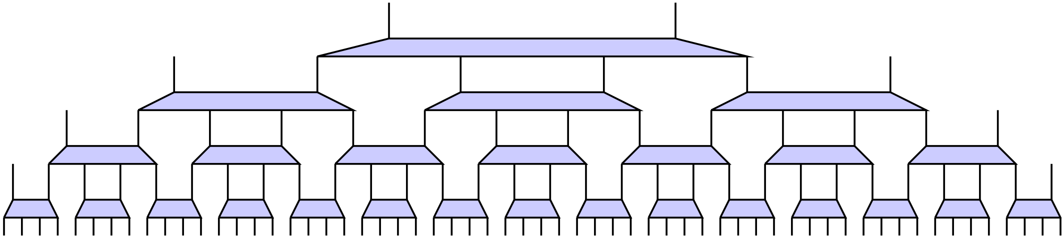
$$=$$



Matrix Product Operator

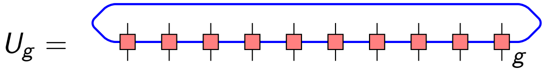
$\mathcal{O} =$ 

Multi-scale Entanglement Renormalization Ansatz

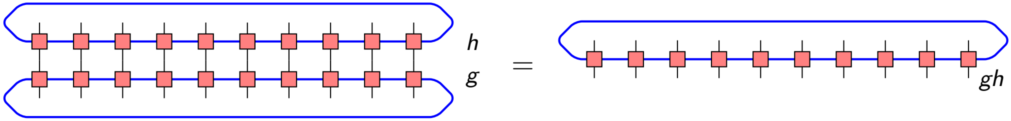


Anomalous MPO Symmetry

MPO symmetry



$$U_g U_h = U_{gh}$$

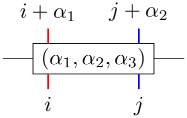


For our example

$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_j Z_{j+1} + \tilde{Z}_j \tilde{Z}_{j+1}) - c\sum(Z_j \tilde{X}_j Z_{j+1} + \tilde{Z}_j X_{j+1} \tilde{Z}_{j+1})$$

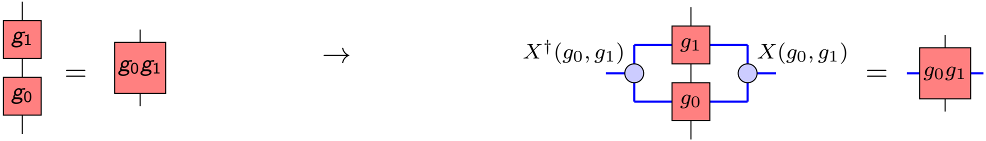
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ MPO symmetry

$$(1,0,0) \mapsto \bigotimes_j X_j, \qquad (0,1,0) \mapsto \bigotimes_j \tilde{X}_j, \qquad (0,0,1) \mapsto \prod_j CZ_{j,j+1}$$



$$= \sum_{k=0}^{1} (-1)^{j\alpha_3(k-i)} |i\rangle\langle k|$$

Third Cohomology Anomaly



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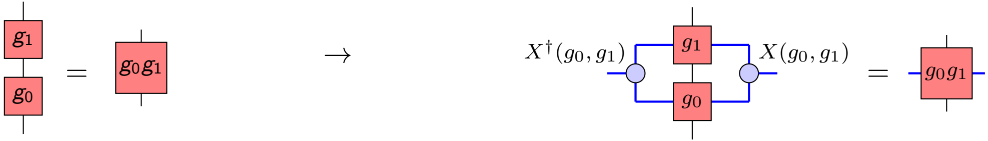

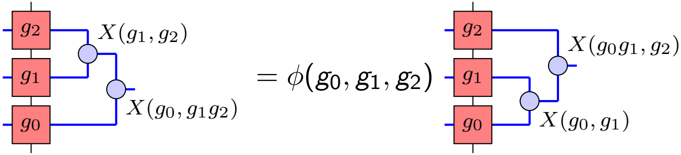

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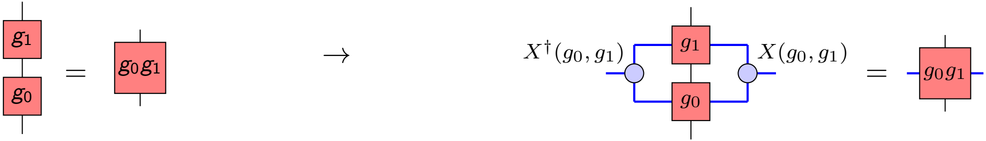
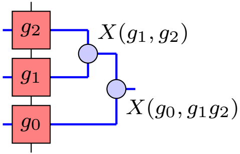
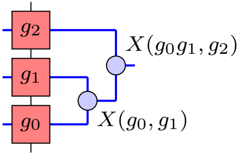


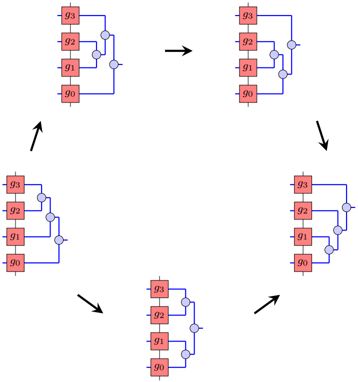


$= \phi(g_0, g_1, g_2)$



$$\phi'(g_0, g_1, g_2) = \phi(g_0, g_1, g_2) \frac{\beta(g_1, g_2)\beta(g_0, g_1 g_2)}{\beta(g_0, g_1)\beta(g_0 g_1, g_2)}$$

Third Cohomology Anomaly

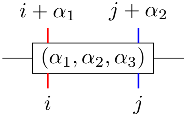


$$\frac{\phi(g_0, g_1, g_2)\phi(g_0, g_1 g_2, g_3)\phi(g_1, g_2, g_3)}{\phi(g_0 g_1, g_2, g_3)\phi(g_0, g_1, g_2 g_3)} = 1$$

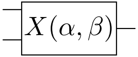
$$\Downarrow$$

$$[\phi] \in \mathcal{H}^3(\mathcal{G}, \mathsf{U}(1))$$

For our Example

MPO tensor: 

$$= \sum_{k=0}^{1} (-1)^{j\alpha_3(k-i)} \, |i\rangle\langle k|$$

Reduction tensor: 

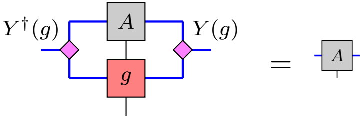
$$= \sum_{x=0}^{1} (-1)^{-x\alpha_2\beta_3} \left| \begin{matrix} x+\alpha_1 \\ x \end{matrix} \right\rangle \langle i|$$

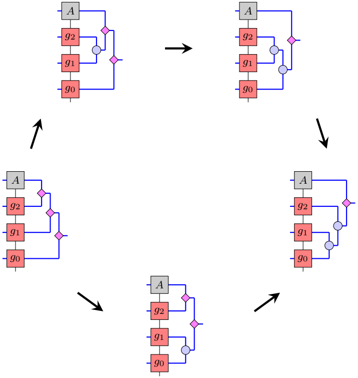
Cocycle: $\phi(\alpha, \beta, \gamma) = (-1)^{\alpha_1\beta_2\gamma_3}$

$$[\phi] \cong (0,0,1) \in \mathcal{H}^3(\mathbb{Z}_2^3, \mathrm{U}(1)) \cong \mathbb{Z}_2^3 \times \mathbb{Z}_2^3 \times \mathbb{Z}_2$$

‘type-III’

No Gapped MPO-Symmetric Groundstates





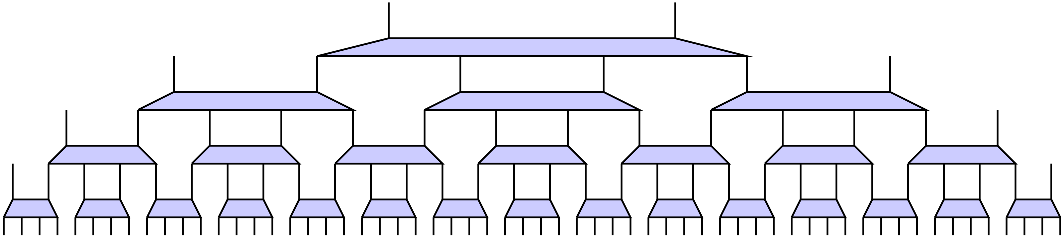
$$\frac{\phi(g_0, g_1, g_2)\pi(g_0, g_1 g_2)\pi(g_1, g_2)}{\pi(g_0 g_1, g_2)\pi(g_0, g_1)} = 1$$

$$\Downarrow$$

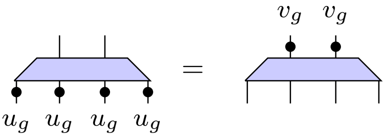
$$[\phi] = [0]$$

MPO Symmetry in MERA

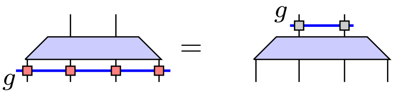
MERA



Symmetry in MERA: on-site vs Anomalous

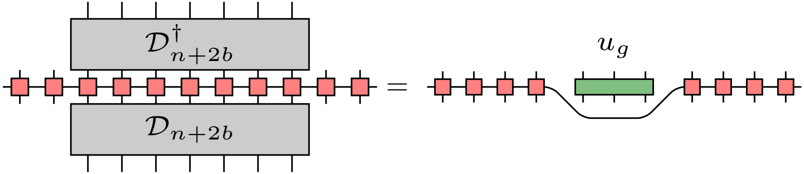


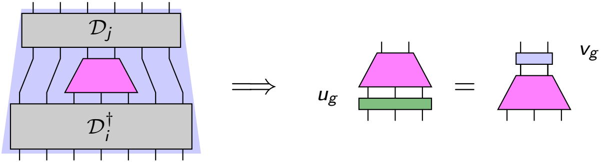
Representation theory



?????

Disentangling an Anomalous Symmetry





For the example

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

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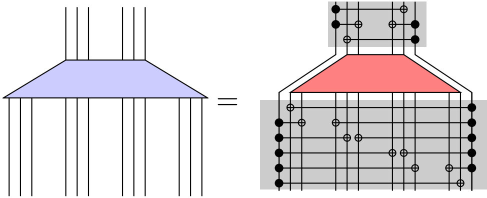
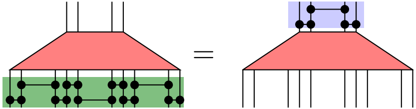
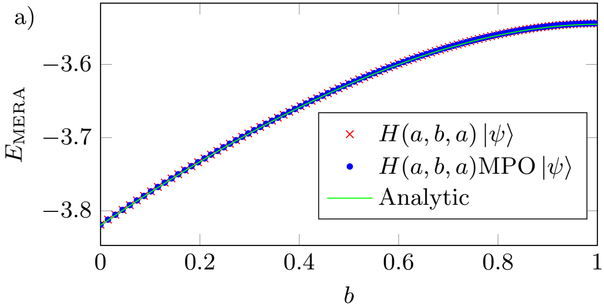




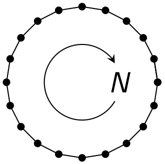
Numerical Results for $a = c$: Energy Density

$$H = -a\sum(X_j + \tilde{X}_j) - b\sum(Z_j Z_{j+1} + \tilde{Z}_j \tilde{Z}_{j+1}) - c\sum(Z_j \tilde{X}_j Z_{j+1} + \tilde{Z}_j X_{j+1} \tilde{Z}_{j+1})$$



Quick aside on conformal field theory

- Field theory describing the low energy physics at a gapless point
- 'Conformal'=Scale invariant (no gap $\implies$ no energy scale)
- Scaling Dimensions
- Conformal spins



Scaling Dimensions

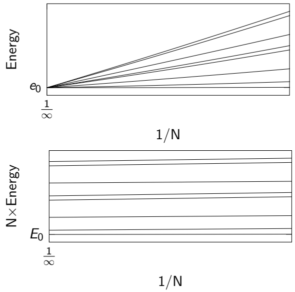
$$\langle \phi_x \phi_y \rangle \sim \frac{1}{|x-y|^{\Delta}}$$

$$\Delta \propto E_n / E_0$$

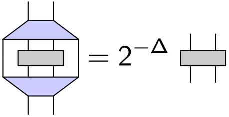
$$\mathcal{S}(\phi) \sim \exp(-\Delta)$$

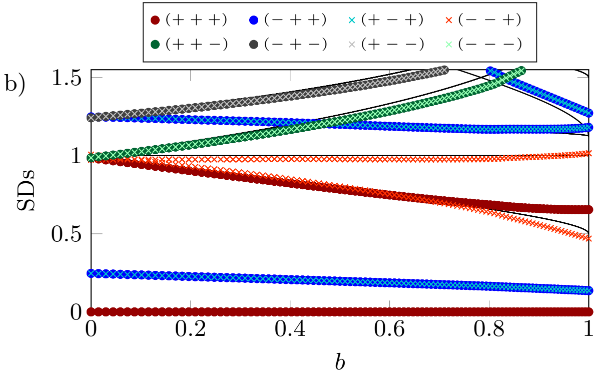
Conformal spins

$$p = \exp\left(\frac{2\pi i}{N} s\right)$$



Numerical Results for $a = c$: Scaling Dimensions





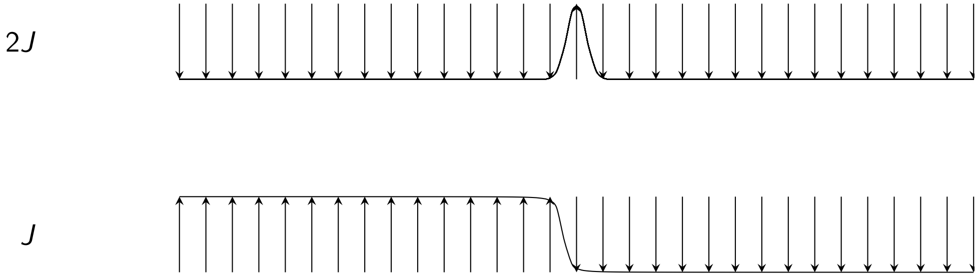
- Colours/symbols indicate irrep
- Compactified boson CFT with radius $R^2 = \frac{\pi}{2\cos^{-1}(\frac{2b}{b-3})}$.
- Fields labeled by $e, m \in \mathbb{Z}$

$$\text{SD: } \Delta_{e,m} = \frac{e^2}{R^2} + \frac{m^2 R^2}{4}$$

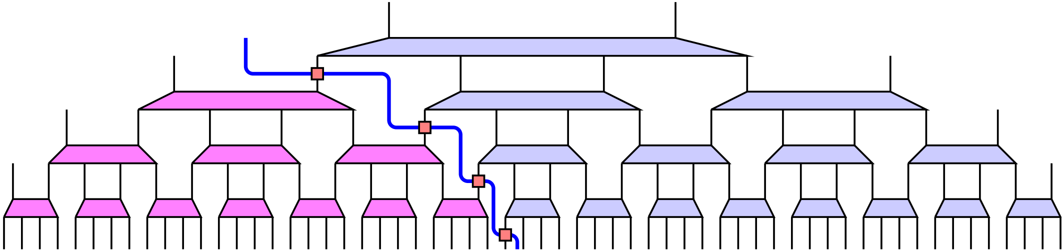
conf. spin: $s_{e,m} = em$

Topological Sectors in MERA

Twisting by a Symmetry

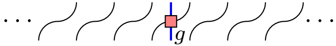


Twisting by an MPO Symmetry

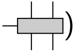
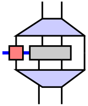


Twisted Translation and Scaling Operators

- Twisted translation operator

$\tau_g :=$ 

- Twisted scaling superoperator

$\mathcal{S}_g($  $) :=$ 

Fractional Conformal Spin

For local fields

$$\tau_1^L = \mathrm{Id}$$

$\implies$ conformal spins are integers

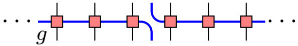
In g twisted sector

$$\tau_g^L = T_g$$

gives a Dehn twist

Fractional Conformal Spin

Dehn twist

$T_g =$ 

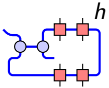
$$T_g^{n_g} = (-1)^{[\phi_g]}\mathrm{Id}$$

$\implies$ conformal spins may lie in $\mathbb{Z} + \frac{1}{n_g}\mathbb{Z}_{n_g} + \frac{[\phi_g]}{n_g^2}$

For our example, some sectors have integers (bosonic), some have $\mathbb{Z} + \frac{1}{2}$ (fermionic) and some $\mathbb{Z} \pm \frac{1}{4}$ ((anti-)semionic)

Projective Symmetry of Twists

g-twisted field transforms under the projective rep

$V_h^{(g)} =$ 

with 2-cocycle given by the slant product of the anomaly label ϕ

For our example

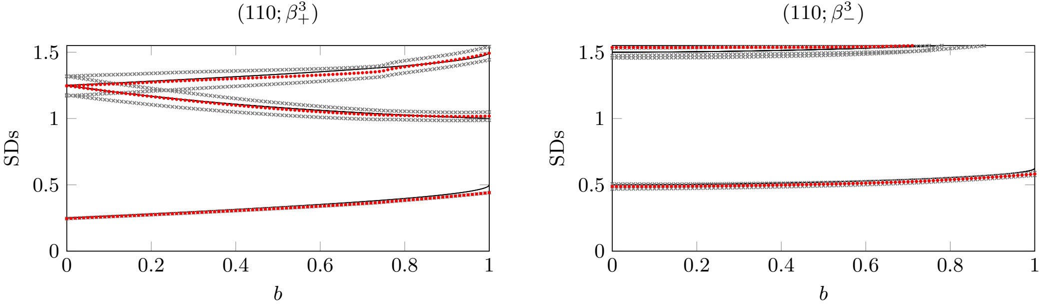
Trivial twist $g = (0,0,0)$: 8 linear irreps.
7 nontrivial twists : 2 projective irreps. each

$$\Downarrow$$

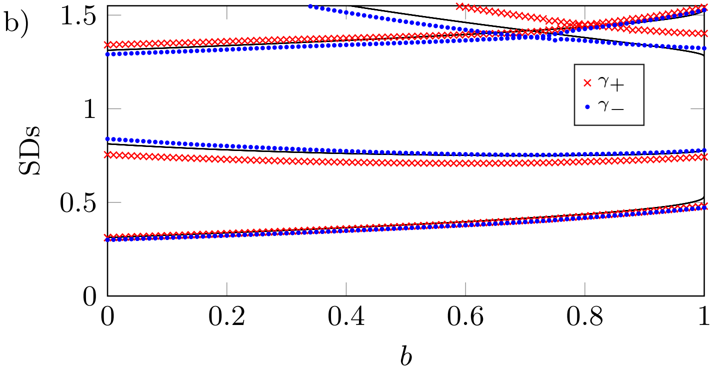
22 irreducible topological sectors

Projective irreps are 2 dimensional $\Longrightarrow$ nonabelian fusion

Numerical Results for $a = c$: Topological Sectors



Numerical Results for $a = c$: Topological Sectors

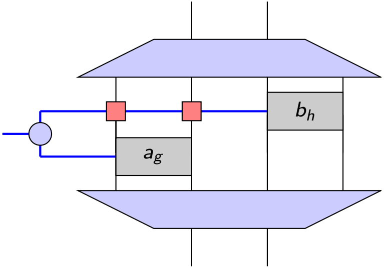


Topological Sectors in Our Example

Topological Sector		Topo. spin	Scal. Dim.	Conf. spin	Parameters	
Twist	Proj. Irrep.					
(110)	β^3_+	0	$\frac{e^2}{R^2} + \frac{m^2 R^2}{4}$	em	$e \in \mathbb{Z} + \frac{1}{2}, m \in \mathbb{Z}$	$em \in \mathbb{Z}$
	β^3_-	$\frac{1}{2}$				$em \in \mathbb{Z} + \frac{1}{2}$
(111)	γ_+	$\frac{3}{4}$	$\frac{e^2}{R^2} + \frac{m^2 R^2}{4}$	em	$e, m \in \mathbb{Z} + \frac{1}{2}$	$em \in \mathbb{Z} + \frac{3}{4}$
	γ_-	$\frac{1}{4}$				$em \in \mathbb{Z} + \frac{1}{4}$

Correlation functions

$\langle a_i b_{i+1} \rangle =$



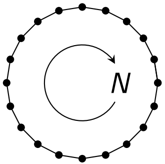
$= \sum_c C_{ab}^c \langle c_i \rangle.$

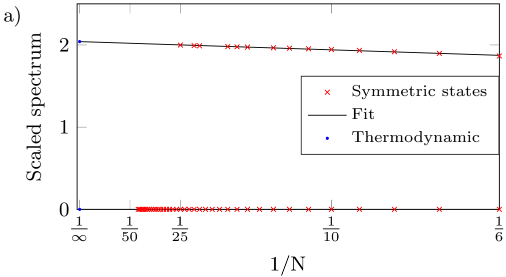
Nonzero C_{ab}^c restricted by topological sectors: c must have twist gh and transform under an appropriate projective irrep.

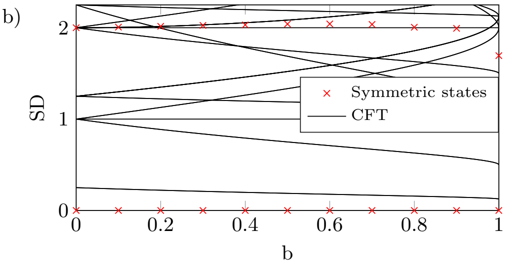
Fusion rules

a × b	χ^1_+	χ^2_+	χ^3_+	χ^4_+	χ^1_-	χ^2_-	χ^3_-	χ^4_-	α^1_+	α^1_-	α^2_+	α^2_-	β^3_+	β^3_-	α^3_+	α^3_-	β^2_+	β^2_-	β^1_+	β^1_-	γ_+	γ_-
χ^1_+	χ^1_+	χ^2_+	χ^3_+	χ^4_+	χ^1_-	χ^2_-	χ^3_-	χ^4_-	α^1_+	α^1_-	α^2_+	α^2_-	β^3_+	β^3_-	α^3_+	α^3_-	β^2_+	β^2_-	β^1_+	β^1_-	γ_+	γ_-
χ^2_+	χ^2_+	χ^1_+	χ^4_+	χ^3_+	χ^2_-	χ^1_-	χ^4_-	χ^3_-	α^1_-	α^1_+	α^2_+	α^2_-	β^3_-	β^3_+	α^3_+	α^3_-	β^2_-	β^2_+	β^1_+	β^1_-	γ_-	γ_+
χ^3_+	χ^3_+	χ^4_+	χ^1_+	χ^2_+	χ^3_-	χ^4_-	χ^1_-	χ^2_-	α^1_+	α^1_-	α^2_-	α^2_+	β^3_-	β^3_+	α^3_+	α^3_-	β^2_+	β^2_-	β^1_-	β^1_+	γ_-	γ_+
χ^4_+	χ^4_+	χ^3_+	χ^2_+	χ^1_+	χ^4_-	χ^3_-	χ^2_-	χ^1_-	α^1_-	α^1_+	α^2_-	α^2_+	β^3_+	β^3_-	α^3_+	α^3_-	β^2_-	β^2_+	β^1_-	β^1_+	γ_+	γ_-
χ^1_-	χ^1_-	χ^2_-	χ^3_-	χ^4_-	χ^1_+	χ^2_+	χ^3_+	χ^4_+	α^1_+	α^1_-	α^2_+	α^2_-	β^3_+	β^3_-	α^3_-	α^3_+	β^2_-	β^2_+	β^1_-	β^1_+	γ_-	γ_+
χ^2_-	χ^2_-	χ^1_-	χ^4_-	χ^3_-	χ^2_+	χ^1_+	χ^4_+	χ^3_+	α^1_-	α^1_+	α^2_+	α^2_-	β^3_-	β^3_+	α^3_-	α^3_+	β^2_+	β^2_-	β^1_-	β^1_+	γ_+	γ_-
χ^3_-	χ^3_-	χ^4_-	χ^1_-	χ^2_-	χ^3_+	χ^4_+	χ^1_+	χ^2_+	α^1_+	α^1_-	α^2_-	α^2_+	β^3_-	β^3_+	α^3_-	α^3_+	β^2_-	β^2_+	β^1_+	β^1_-	γ_+	γ_-
χ^4_-	χ^4_-	χ^3_-	χ^2_-	χ^1_-	χ^4_+	χ^3_+	χ^2_+	χ^1_+	α^1_-	α^1_+	α^2_-	α^2_+	β^3_+	β^3_-	α^3_-	α^3_+	β^2_+	β^2_-	β^1_+	β^1_-	γ_-	γ_+
α^1_+	α^1_+	α^1_-	α^1_+	α^1_-	α^1_+	α^1_-	α^1_+	α^1_-	χ^1_+ χ^3_+ χ^1_- χ^3_-	χ^2_+ χ^4_+ χ^2_- χ^4_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-
α^1_-	α^1_-	α^1_+	α^1_-	α^1_+	α^1_-	α^1_+	α^1_-	α^1_+	χ^2_+ χ^4_+ χ^2_- χ^4_-	χ^1_+ χ^3_+ χ^1_- χ^3_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-
α^2_+	α^2_+	α^2_+	α^2_-	α^2_-	α^2_+	α^2_+	α^2_-	α^2_-	β^3_+ β^3_-	β^3_+ β^3_-	χ^1_+ χ^2_+ χ^1_- χ^2_-	χ^3_+ χ^4_+ χ^3_- χ^4_-	α^1_+ α^1_-	α^1_+ α^1_-	β^1_+ β^1_-	β^1_+ β^1_-	γ_+ γ_-	γ_+ γ_-	α^3_+ α^3_-	α^3_+ α^3_-	β^2_+ β^2_-	β^2_+ β^2_-
α^2_-	α^2_-	α^2_-	α^2_+	α^2_+	α^2_-	α^2_-	α^2_+	α^2_+	β^3_+ β^3_-	β^3_+ β^3_-	χ^3_+ χ^4_+ χ^3_- χ^4_-	χ^1_+ χ^2_+ χ^1_- χ^2_-	α^1_+ α^1_-	α^1_+ α^1_-	β^1_+ β^1_-	β^1_+ β^1_-	γ_+ γ_-	γ_+ γ_-	α^3_+ α^3_-	α^3_+ α^3_-	β^2_+ β^2_-	β^2_+ β^2_-
β^3_+	β^3_+	β^3_-	β^3_-	β^3_+	β^3_+	β^3_-	β^3_-	β^3_+	α^2_+ α^2_-	α^2_+ α^2_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^1_+ χ^4_+ χ^1_- χ^4_-	χ^2_+ χ^3_+ χ^2_- χ^3_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-
β^3_-	β^3_-	β^3_+	β^3_+	β^3_-	β^3_-	β^3_+	β^3_+	β^3_-	α^2_+ α^2_-	α^2_+ α^2_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^2_+ χ^3_+ χ^2_- χ^3_-	χ^1_+ χ^4_+ χ^1_- χ^4_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-
α^3_+	α^3_+	α^3_+	α^3_+	α^3_+	α^3_-	α^3_-	α^3_-	α^3_-	β^2_+ β^2_-	β^2_+ β^2_-	β^1_+ β^1_-	β^1_+ β^1_-	γ_+ γ_-	γ_+ γ_-	χ^1_+ χ^2_+ χ^3_+ χ^4_+	χ^1_- χ^2_- χ^3_- χ^4_-	α^1_+ α^1_-	α^1_+ α^1_-	α^2_+ α^2_-	α^2_+ α^2_-	β^3_+ β^3_-	β^3_+ β^3_-
α^3_-	α^3_-	α^3_-	α^3_-	α^3_-	α^3_+	α^3_+	α^3_+	α^3_+	β^2_+ β^2_-	β^2_+ β^2_-	β^1_+ β^1_-	β^1_+ β^1_-	γ_+ γ_-	γ_+ γ_-	χ^1_- χ^2_- χ^3_- χ^4_-	χ^1_+ χ^2_+ χ^3_+ χ^4_+	α^1_+ α^1_-	α^1_+ α^1_-	α^2_+ α^2_-	α^2_+ α^2_-	β^3_+ β^3_-	β^3_+ β^3_-
β^2_+	β^2_+	β^2_-	β^2_+	β^2_-	β^2_-	β^2_+	β^2_-	β^2_+	α^3_+ α^3_-	α^3_+ α^3_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^2_+ χ^4_+ χ^1_- χ^3_-	χ^1_+ χ^3_+ χ^2_- χ^4_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-
β^2_-	β^2_-	β^2_+	β^2_-	β^2_+	β^2_+	β^2_-	β^2_+	β^2_-	α^3_+ α^3_-	α^3_+ α^3_-	γ_+ γ_-	γ_+ γ_-	β^1_+ β^1_-	β^1_+ β^1_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^1_+ χ^3_+ χ^2_- χ^4_-	χ^2_+ χ^4_+ χ^1_- χ^3_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-
β^1_+	β^1_+	β^1_+	β^1_-	β^1_-	β^1_-	β^1_-	β^1_+	β^1_+	γ_+ γ_-	γ_+ γ_-	α^3_+ α^3_-	α^3_+ α^3_-	β^2_+ β^2_-	β^2_+ β^2_-	α^2_+ α^2_-	α^2_+ α^2_-	β^3_+ β^3_-	β^3_+ β^3_-	χ^3_+ χ^4_+ χ^1_- χ^2_-	χ^1_+ χ^2_+ χ^3_- χ^4_-	α^1_+ α^1_-	α^1_+ α^1_-
β^1_-	β^1_-	β^1_-	β^1_+	β^1_+	β^1_+	β^1_+	β^1_-	β^1_-	γ_+ γ_-	γ_+ γ_-	α^3_+ α^3_-	α^3_+ α^3_-	β^2_+ β^2_-	β^2_+ β^2_-	α^2_+ α^2_-	α^2_+ α^2_-	β^3_+ β^3_-	β^3_+ β^3_-	χ^1_+ χ^2_+ χ^3_- χ^4_-	χ^3_+ χ^4_+ χ^1_- χ^2_-	α^1_+ α^1_-	α^1_+ α^1_-
γ_+	γ_+	γ_-	γ_-	γ_+	γ_-	γ_+	γ_+	γ_-	β^1_+ β^1_-	β^1_+ β^1_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^1_+ χ^4_+ χ^2_- χ^3_-	χ^2_+ χ^3_+ χ^1_- χ^4_-
γ_-	γ_-	γ_+	γ_+	γ_-	γ_+	γ_-	γ_-	γ_+	β^1_+ β^1_-	β^1_+ β^1_-	β^2_+ β^2_-	β^2_+ β^2_-	α^3_+ α^3_-	α^3_+ α^3_-	β^3_+ β^3_-	β^3_+ β^3_-	α^2_+ α^2_-	α^2_+ α^2_-	α^1_+ α^1_-	α^1_+ α^1_-	χ^2_+ χ^3_+ χ^1_- χ^4_-	χ^1_+ χ^4_+ χ^2_- χ^3_-

Numerical Results for $a = c$: Protected Criticality







Conclusions

- MERA can support anomalous symmetry
  - 1D critical theories at self dual points
  - Gapless edge of 2D SPT phases

- Full decomposition into topological sectors possible
  - Full topological data can be extracted
  - Scaling fields transform projectively
  - Constrains correlation functions

Future Directions

- Looking at a non group examples such as Ising duality
- Translations?
- Tensor networks for anomalous boundary theories of 3D systems

Questions?

arXiv:1703.07782

Numerical Results for $a = b \leftrightarrow c = b$ Duality: Energy + SDs