MERA for Spin Chains with Critical Lines

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ARC Centre for Engineered Quantum Systems,
The University of Sydney, Australia

January 16, 2013

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Overview

- Developing numerical methods to study 1D critical systems
  - Variational algorithm to optimise a MERA description of the ground state

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  - Variational algorithm to optimise a MERA description of the ground state
- Ashkin-Teller model
  - Believed to be described by $c = 1$ CFT with continuously varying critical indices
- Extract physical information about the model of interest
  - Output of our algorithm is consistent with a conformal field theory conjectured to describe the thermodynamic limit of the spin models examined

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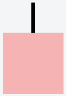
Tensor Network Formalism

$x^{\mu} =$ 

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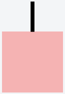
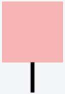
$x^{\mu} =$ 

$x_{\mu} =$ 

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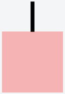
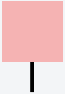
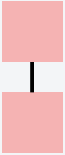
$x^\mu =$ 

$x_\mu =$ 

$x^\mu x_\mu =$ 

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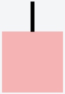
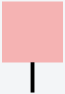
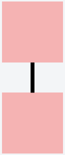
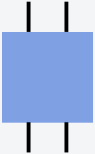
$x^\mu =$ 

$x_\mu =$ 

$x^\mu x_\mu =$ 

$u_{\alpha\beta}^{\gamma\delta} =$ 

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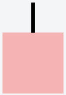
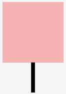
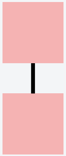
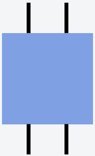
$x^{\mu} =$ 

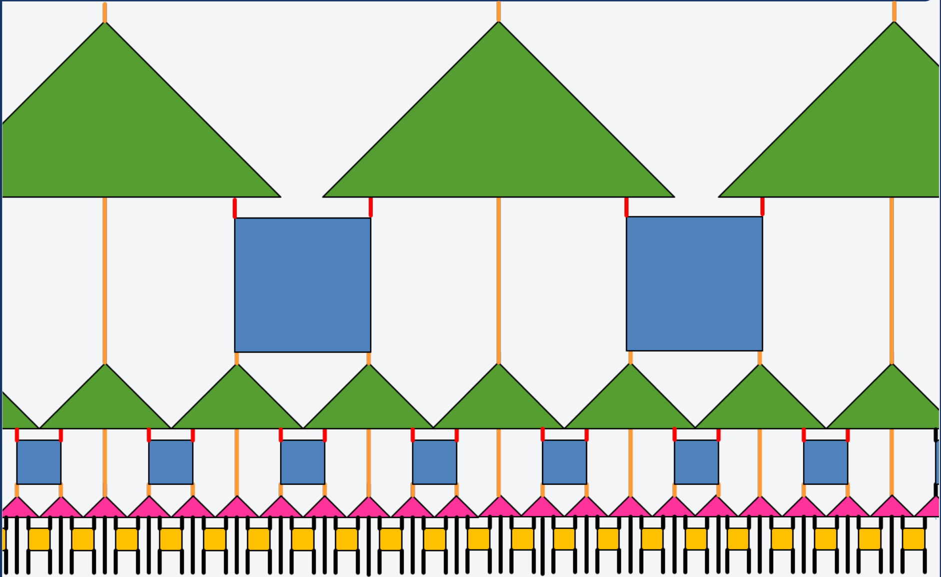
$x_{\mu} =$ 

$x^{\mu} x_{\mu} =$ 

$u_{\alpha\beta}^{\gamma\delta} =$ 

$w_{\alpha\beta\gamma}^{\delta} =$ 

Multiscale Entanglement Renormalization Ansatz



G. Vidal, Physical Review Letters 99, 220405 (2007).

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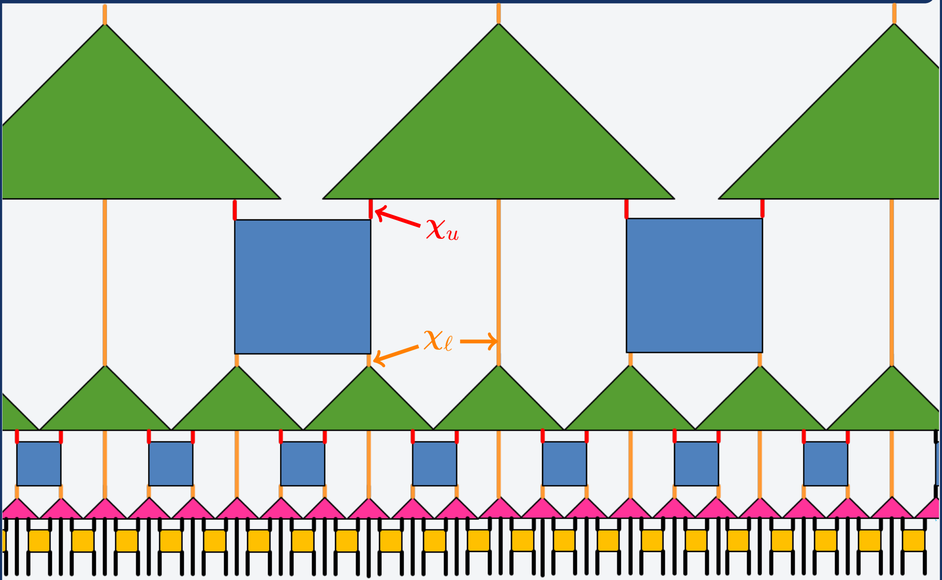
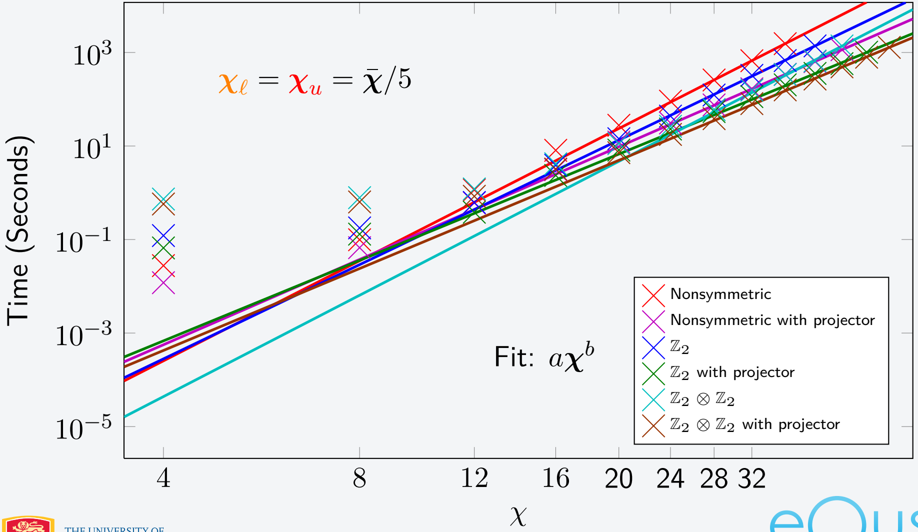


G. Evenbly et al, Physical Review B 82, 132411 (2010).

Algorithm

- G. Evenbly and G. Vidal, Physical Review B 79, 144108 (2009).
- G. Evenbly and G. Vidal, (2011), arXiv:1109.5334v1 [quant-ph].
- R. N. C. Pfeifer, Simulation of Anyons Using Symmetric Tensor Network Algorithms, PhD Thesis ,The University of Queensland, 2011.
- G. Evenbly, Foundations and Applications of Entanglement Renormalization, PhD Thesis, The University of Queensland, 2010.
- S. Singh, R. Pfeifer, and G. Vidal, Physical Review A 82, 050301 (2010).

Scaling of Algorithm



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3.46 GHz Dual Core, 192 Gb RAM

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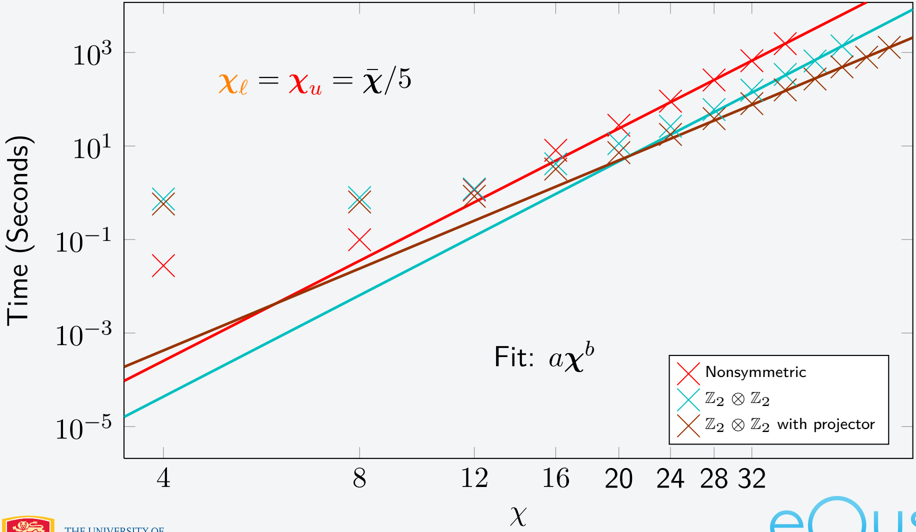

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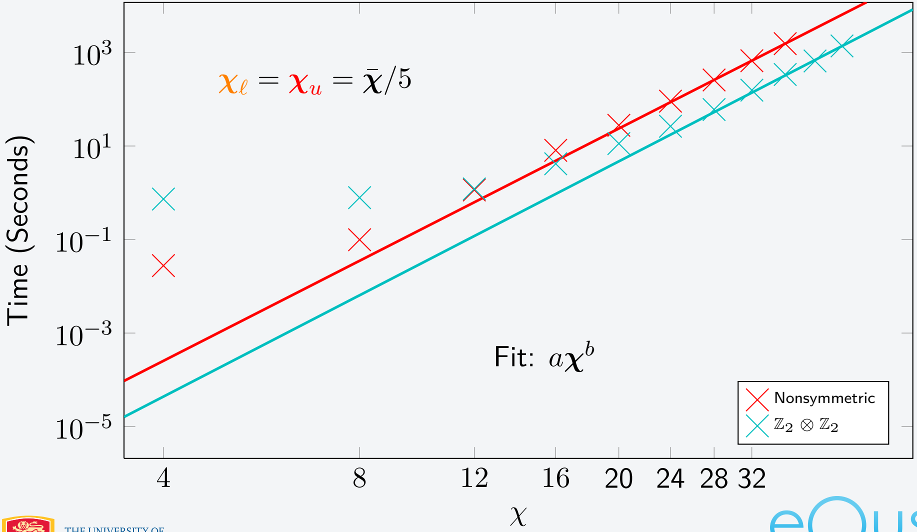

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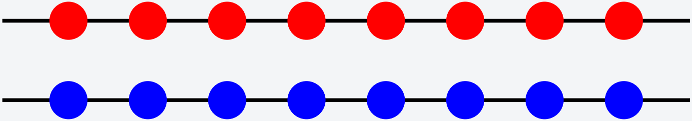
Ashkin-Teller Model

$$H = -\sum_{j=1}^{N} \left(Z_j + Z_j \qquad\qquad \right)$$

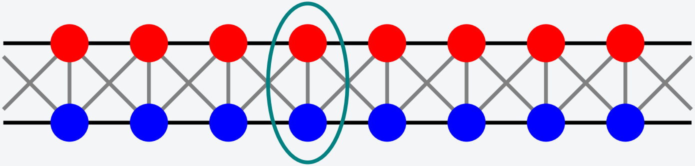
$$- \beta \sum_{j=1}^{N-1} \left(X_j X_{j+1} + X_j X_{j+1} \qquad\qquad\qquad\qquad \right)$$



J. Ashkin and E. Teller, Physical Review 64, 178 (1943).
J. Sólyom, Physical Review B 24, 230 (1981).

Ashkin-Teller Model

$$H_{\mathrm{AT}} = -\sum_{j=1}^{N} \left(Z_j + Z_j + \lambda Z_j Z_j \right) - \beta \sum_{j=1}^{N-1} \left(X_j X_{j+1} + X_j X_{j+1} + \lambda X_j X_j X_{j+1} X_{j+1} \right)$$



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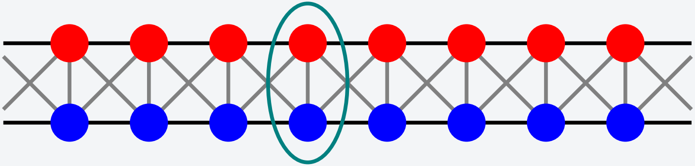
$$S_1 = \prod_{j=1}^{N} Z_j \qquad S_2 = \prod_{j=1}^{N} Z_j$$



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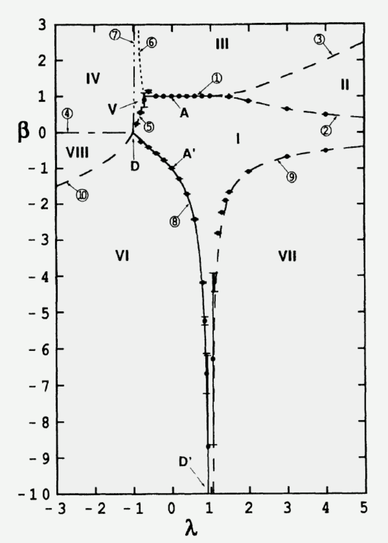
Ashkin-Teller Phase Diagram



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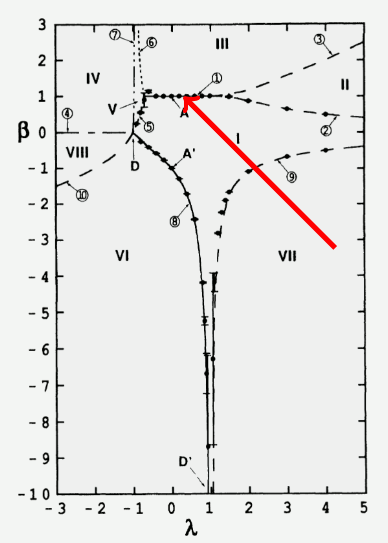

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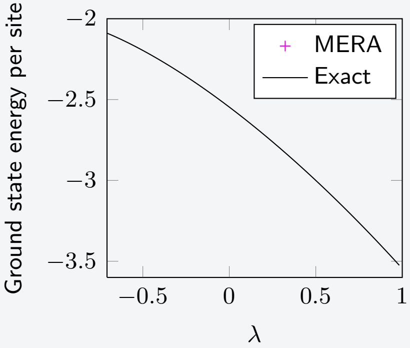
Ashkin-Teller Ground State Energy



$\chi_\ell = 12, \chi_u = 8$

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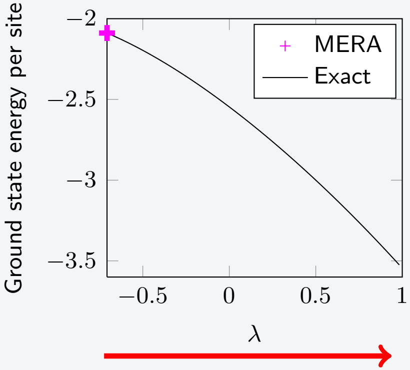

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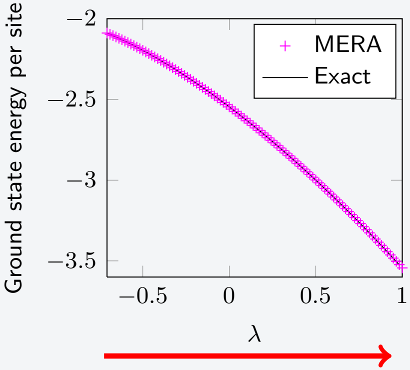

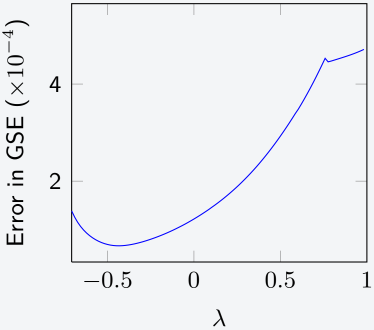

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Conformal Data

- Thermodynamic limit of critical spin chain described by a conformal field theory
- Central charge c
  conformal exponents $h + \bar{h} = \Delta$
  OPE coefficients

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  $c = 1$
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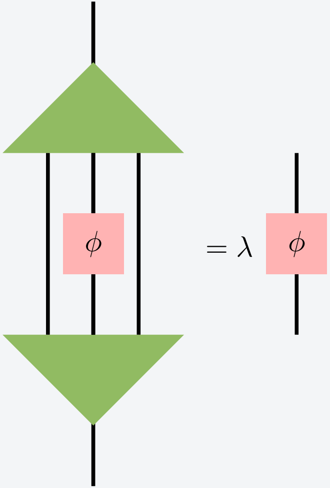
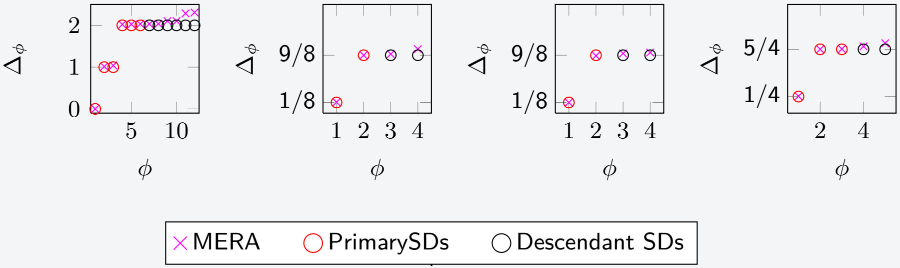


Ashkin-Teller Scaling Dimensions

$\lambda = 0$ $\quad\quad$ $\chi_\ell = 28 = \bar{\chi}/5, \chi_u = 12$



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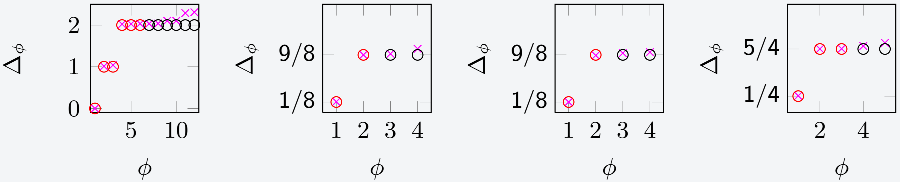
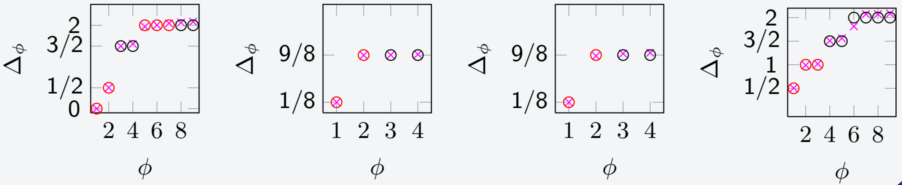
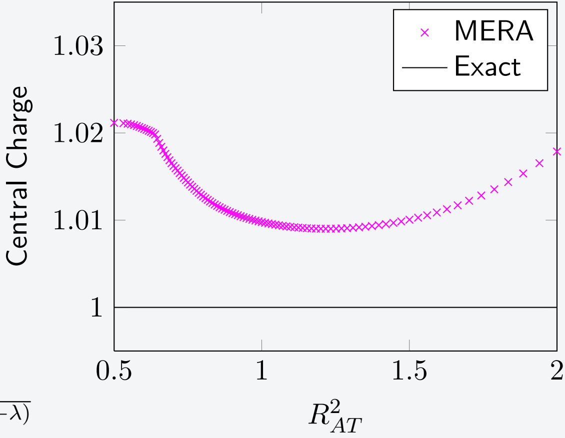


× MERA $\quad$ ○ PrimarySDs $\quad$ ○ Descendant SDs

$\lambda = -\sqrt{2}/2$ $\quad$ $\chi_\ell = 36 = \bar{\chi}/5, \chi_u = 20$



Ashkin-Teller Central Charge



$$R^2_{AT} = \frac{\pi}{2\cos^{-1}(-\lambda)}$$

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$$\chi_\ell = 12, \chi_u = 8$$

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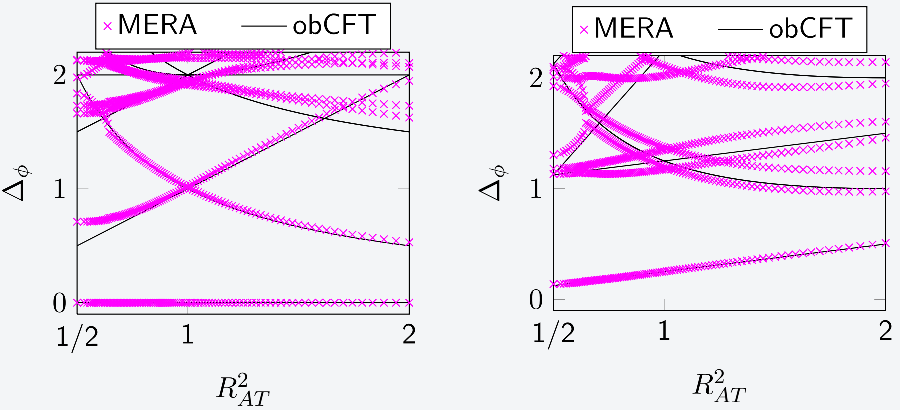
Ashkin-Teller Continuously Varying Exponents



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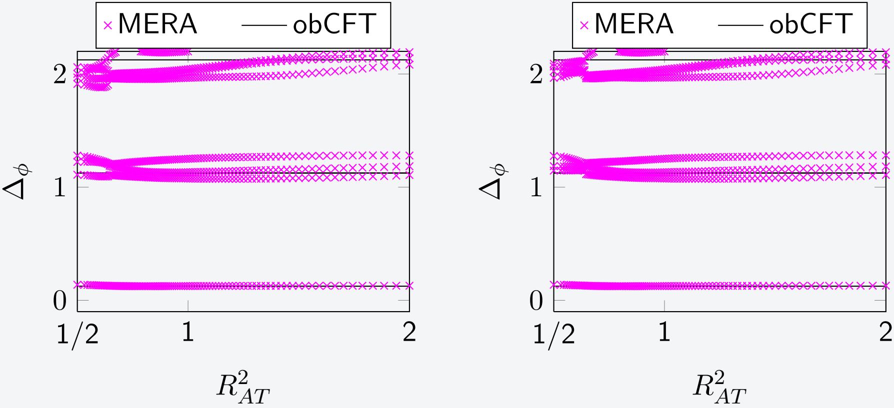

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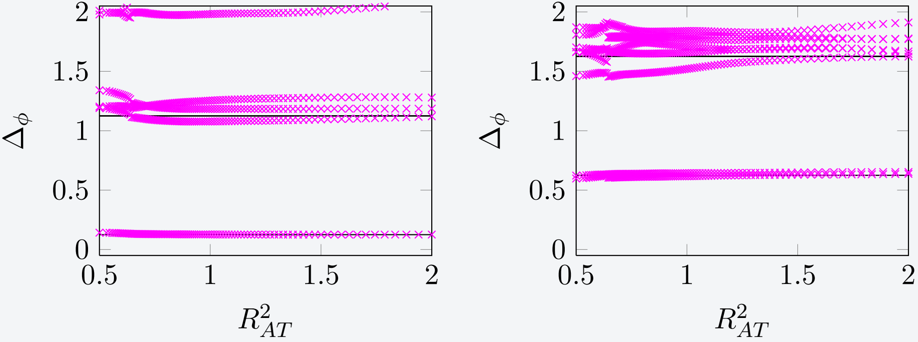
Ashkin-Teller Nonlocal/Twisted



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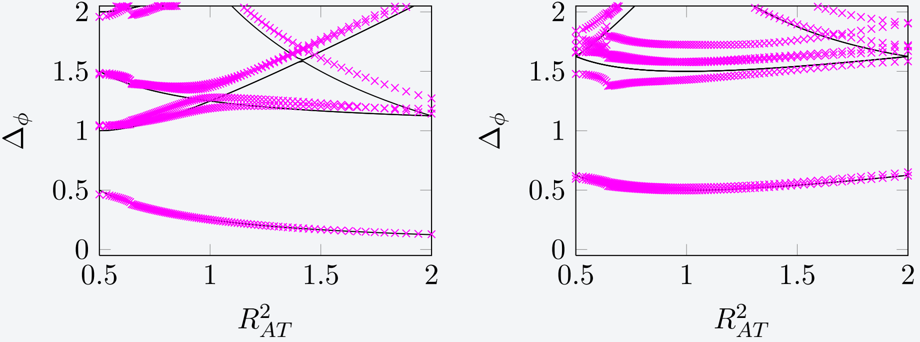

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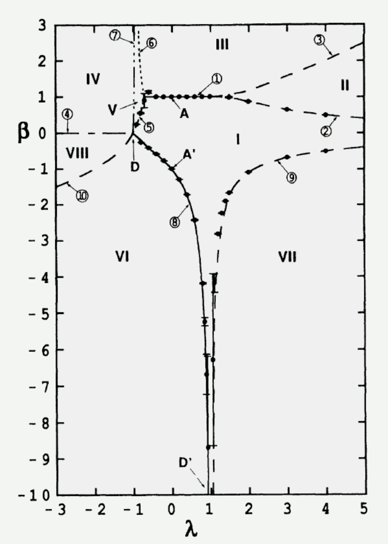
Ashkin-Teller Ising Line



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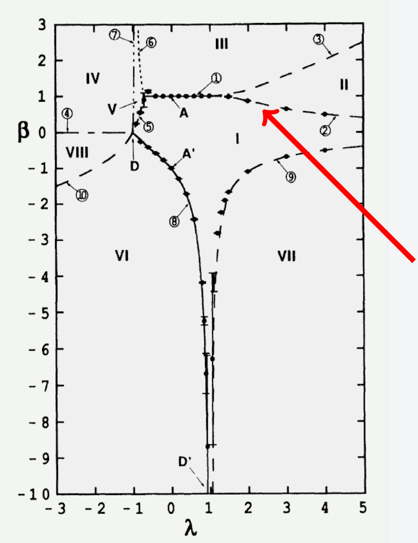

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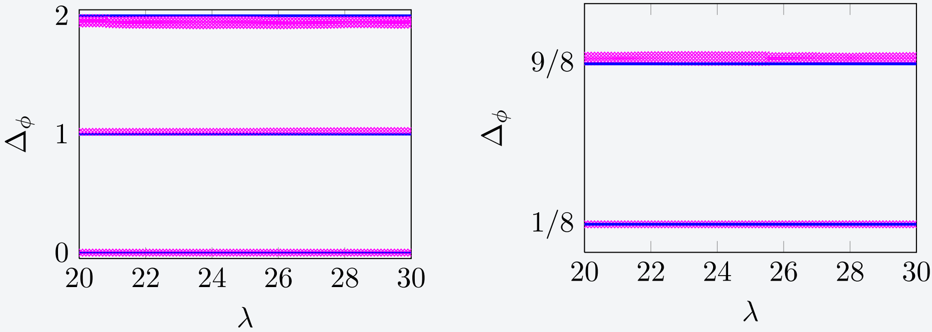
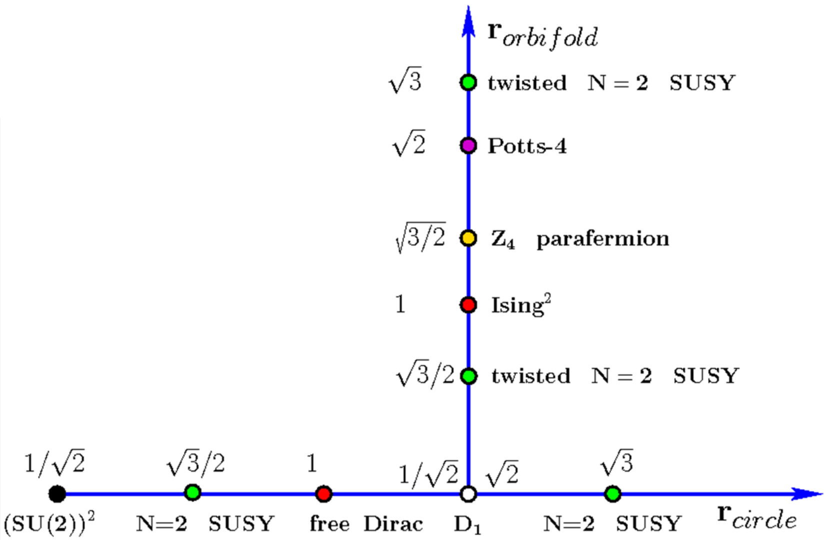


$$\chi_\ell = 16 = \bar{\chi}/4, \chi_u = 12$$

$c = 1$ CFTs



V. Alba, L. Tagliacozzo, P. Calabrese, J. Stat. Mech P06012 (2011)

Conclusions

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- Obtained conformal data for perturbed cluster state consistent with the free boson CFT

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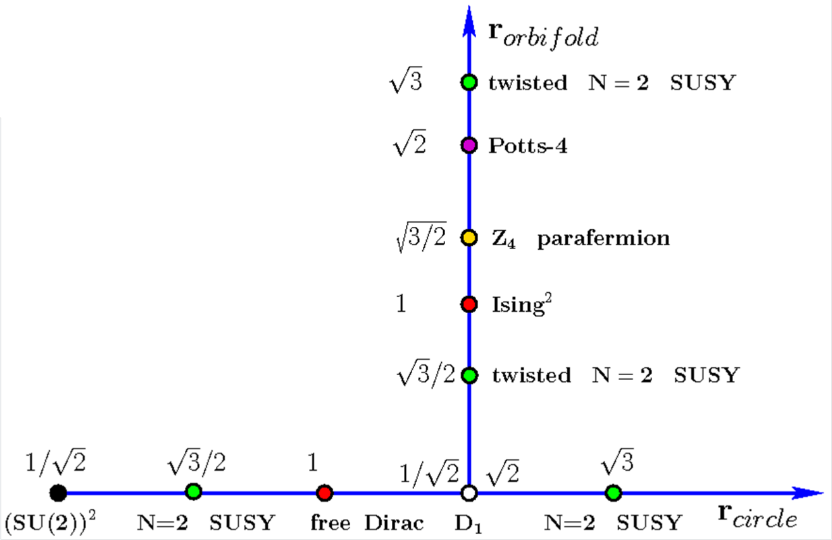
- Incorporated Abelian symmetries present in the models
- Obtained conformal data for Ashkin-Teller consistent with the orbifold boson CFT
- Obtained conformal data for perturbed cluster state consistent with the free boson CFT
- Demonstrated a critical line which does not have continuously varying critical indices

$c = 1$ CFTs



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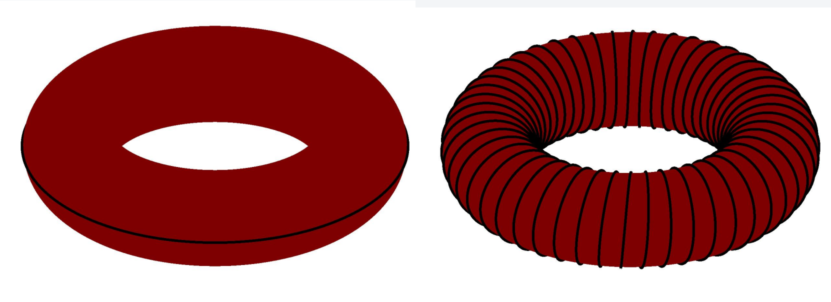
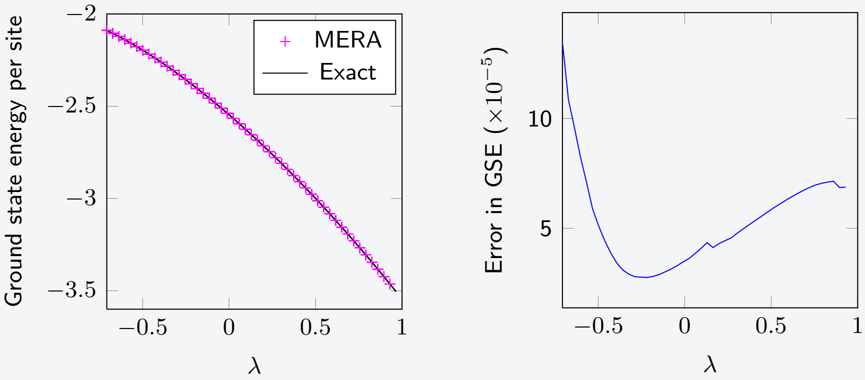
S^1 Boson CFT



Perturbed Cluster State Ground State Energy



$$\chi_\ell = \chi_u = \bar{\chi}/4 = 20$$

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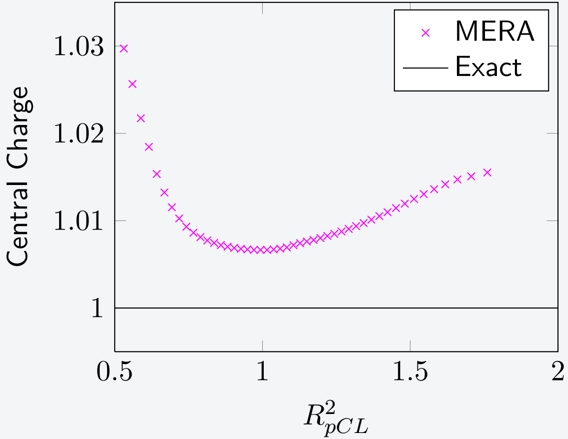
Perturbed Cluster State Central Charge



$$R^2_{pCL} = \frac{2}{\pi}(\pi - \cos^{-1}(\lambda))$$

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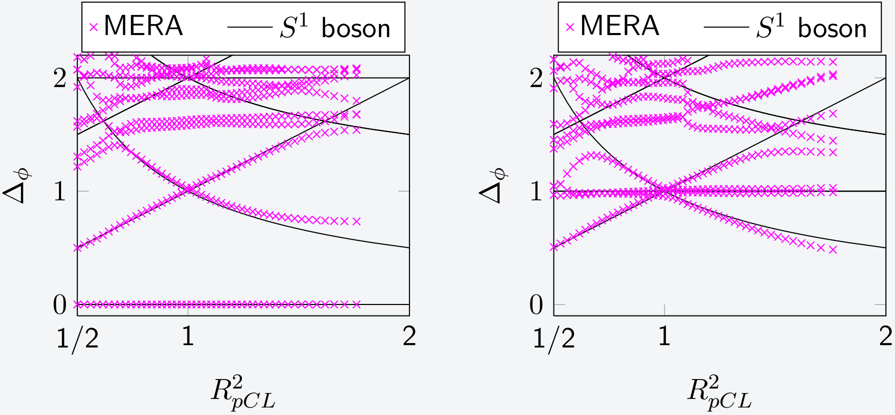
pCL Continuously Varying Exponents



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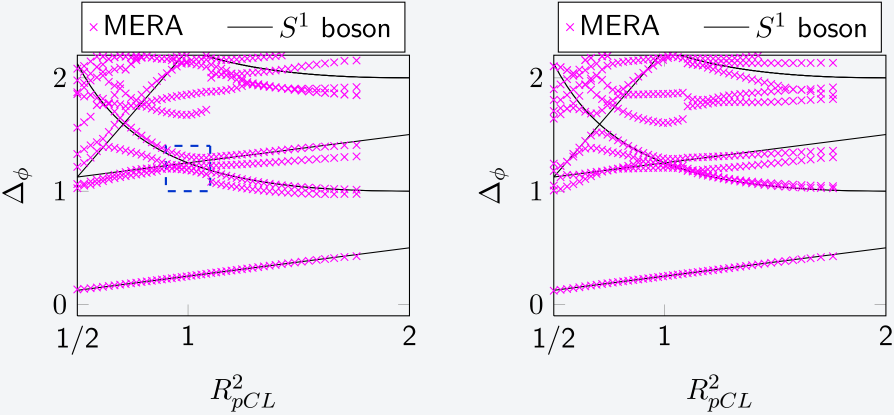

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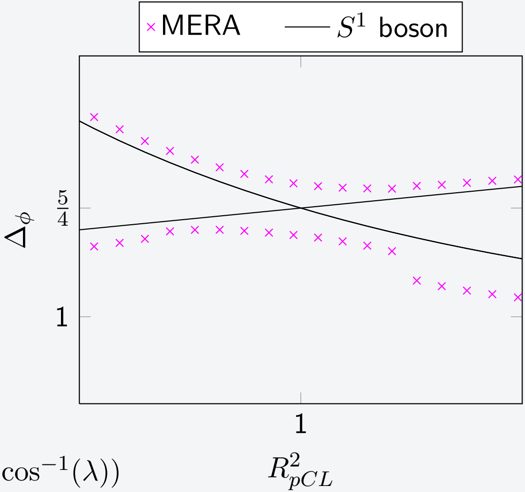
Avoided Crossing



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