Ribbon Operators in Topologically Ordered 2D Spin Systems

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Introduction

- Topologically ordered systems interesting for both condensed matter and quantum computing applications.
- Anyonic excitations associated to topological phase. Created by string-like operators.
- Topological entanglement entropy identifies topological order but does not identify which order (which anyon model)[1].
- We aim to numerically find anyon creation operators away from exactly solvable models.

2D Topologically Ordered Systems

- Long range ordered but no local order parameter. Distinguished by extensive logical operators.
- Ground states can be used to encode quantum information which is stable under local noise.
- Ground state degeneracy depends on topology of lattice.
- Topological phases are robust to local perturbations. Anyon braiding, fusion etc. should be preserved[2].
- Ribbon operators provide a signal of a topological phase, telling us more than just the total quantum dimension.

What makes a good ribbon?

- Use the Toric Code as inspiration.
- Should preserve ground space
  - $[R, H] = 0$.
- Should define logical qubit (Z_2 order) or similar condition.
  - $\{R, \overline{Z}\} = 0$
- Bounded width but extensive length: string-like RG fixed point.
- Should act as a unitary, at least on the ground space. Logical operators map ground states to ground states.
- Design a cost function quantifying these conditions. Numerically optimise over some class of operators.
- How to generalise to the case of no known string?

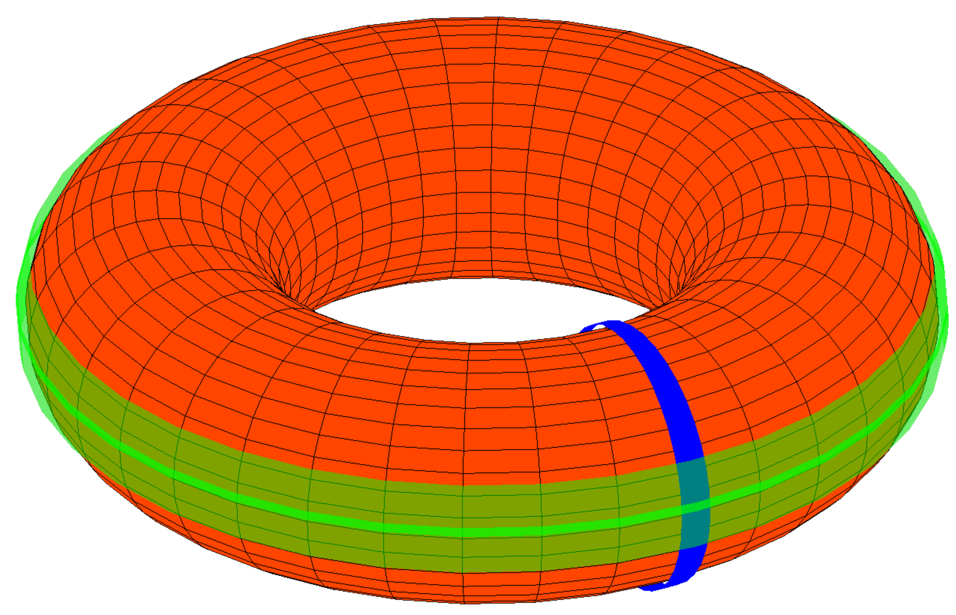
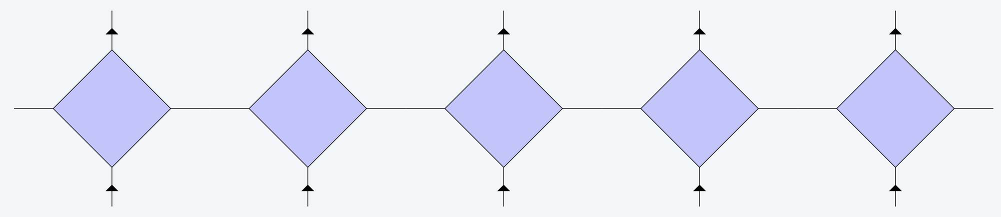


Figure 1: When local perturbations are added to the Toric Code Hamiltonian, we expect the string operators to blur out into ribbons with support on a wider region.

Matrix Product Operators



- Efficient description of 'low entanglement' operators[3].
- Ribbons create particle pairs $\implies$ low entanglement.
- We search for ribbons within the class of translationally invariant, low bond dimension MPOs.

References

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[6] A. Kitaev, Ann. Phys., **321**, 2-11 (2006), arXiv:cond-mat/0506438.

Toric Code

- Exactly solvable model. Logical operators are known strings[4].
- Good for benchmarking numerical technique.
- Phase robust to $\lambda \sum Z$ for large λ[5].
- Can we use the ribbon operators as an order parameter to detect the end of the topological phase?

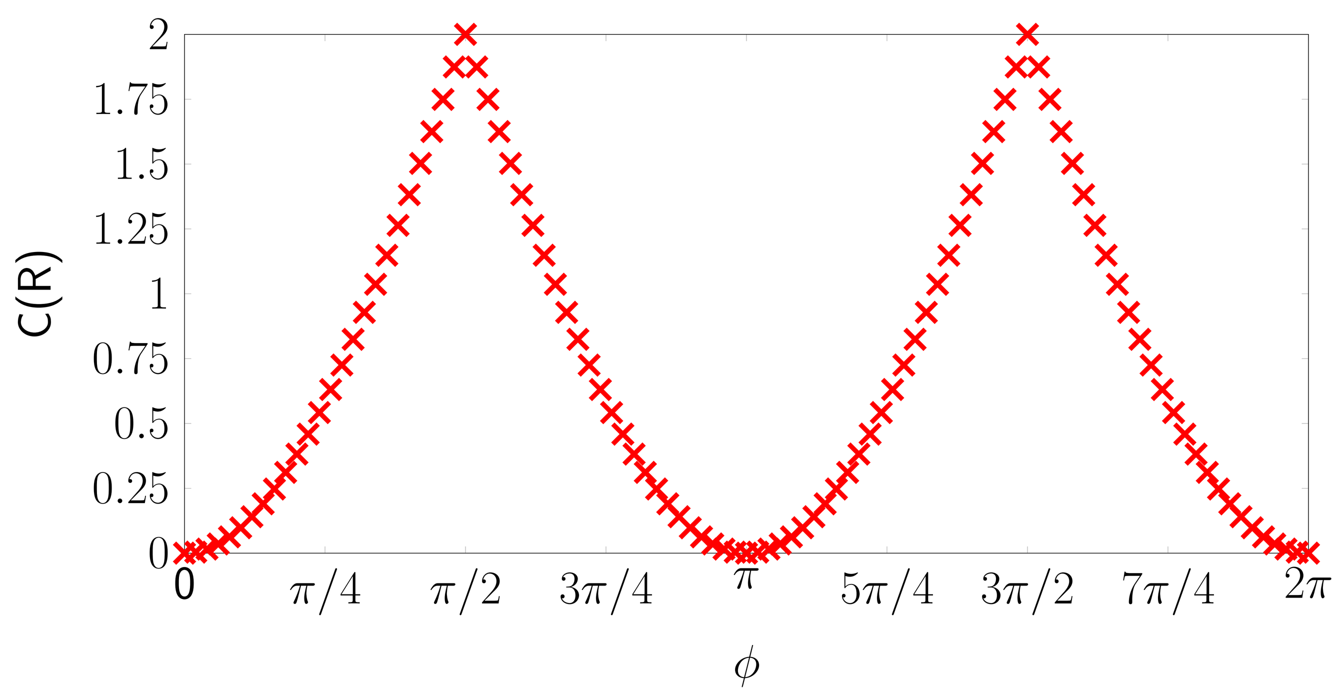

Figure 2: 'Cost' of a ribbon operator in the Toric code when we ask for $R\overline{Z} = e^{i\phi}\overline{Z}R$ rather than anticommuting. We see a strong dip at $\phi = \pi$.

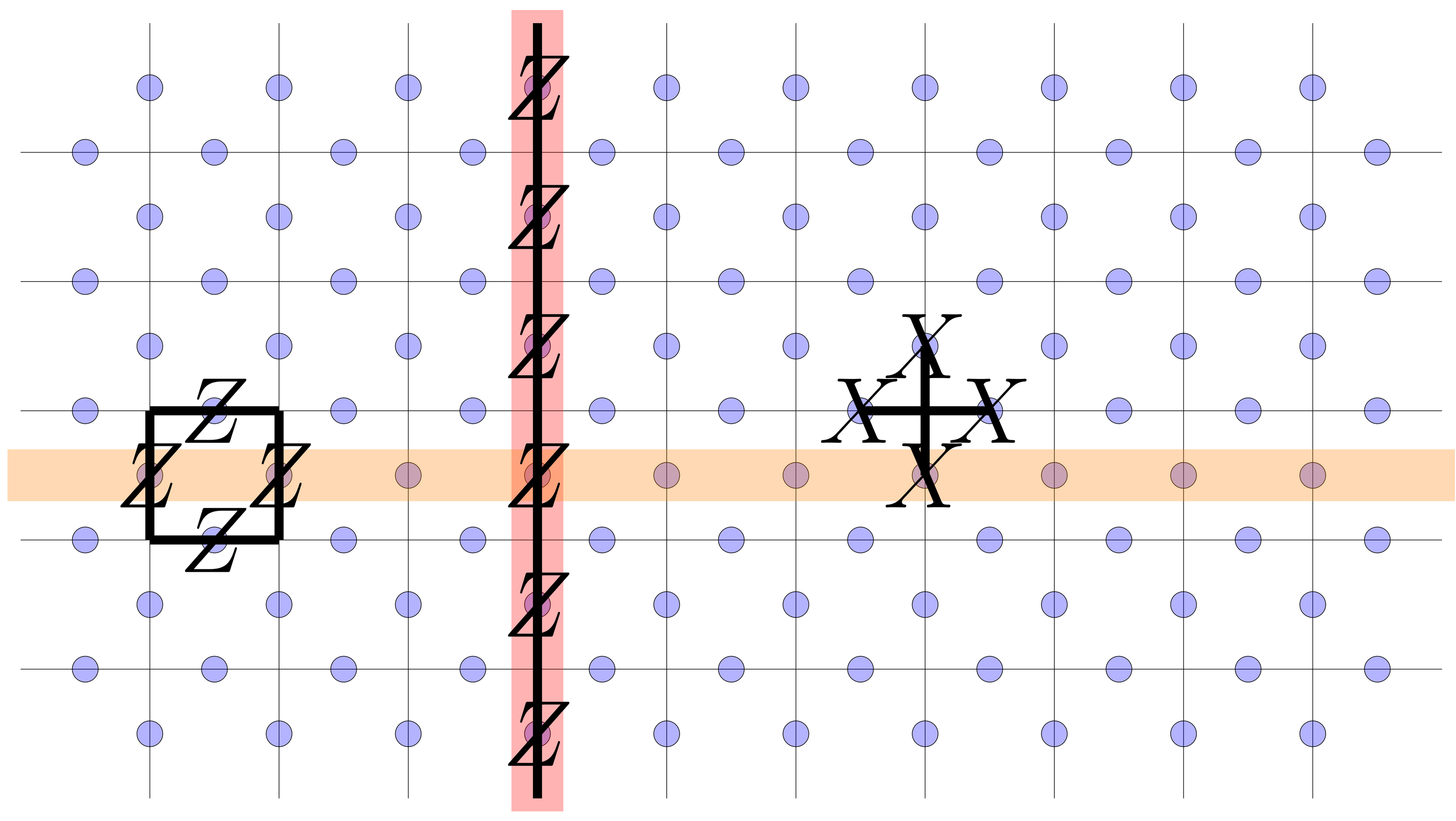

Figure 3: Searching for a width 1 ribbon (string) operator for the Toric code. We need to consider the dual string $\prod Z$ shown in red and the interior parts of the Hamiltonian terms which cross the boundary (sample star and plaquette indicated).

Kitaev's Honeycomb Model

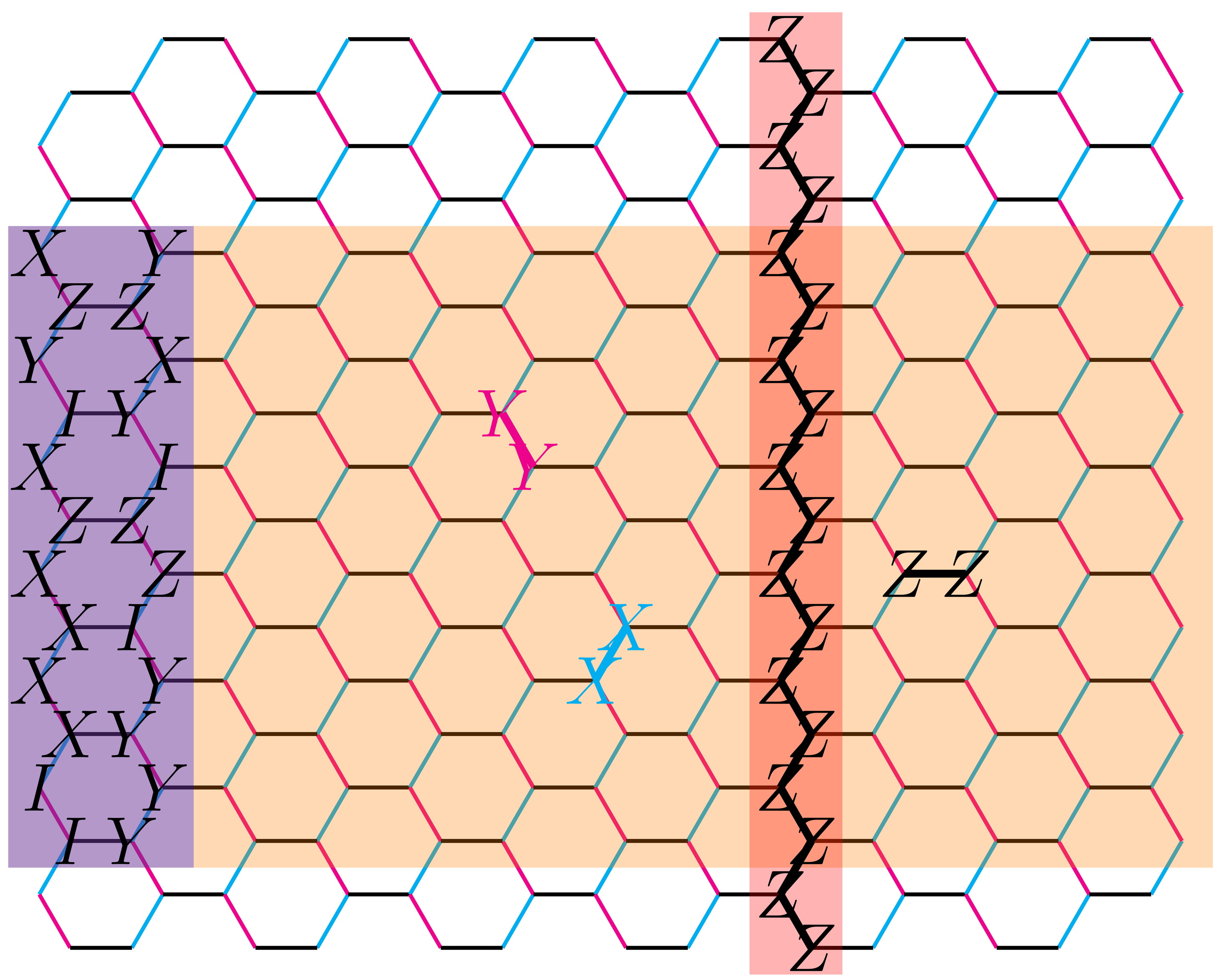

Figure 4: Kitaev's Honeycomb model; qubits sit on the vertices of a honeycomb lattice. The Hamiltonian consists of $-J_x \sum XX$ along all cyan links, $-J_y \sum YY$ along magenta links and $-J_z \sum ZZ$ along the black links (examples of each shown). One ribbon operator ($w = 1$) consists of a product of Z as shown. The other is not known. We define the unit cell of the ribbon as shown in blue, with MPO tensors numbered as indicated found using our algorithm for a width 12 ribbon at $J_x = J_y = J_z/10$.

- Consists of qubits on a honeycomb lattice with three kinds of interactions[6].
- Has phase supporting Z_2 (Toric code) order.
- Also has a (gapless) phase supporting nonabelian anyons.
- What do the ribbon operators look like in the Z_2 phase?
- What happens at the phase transition?

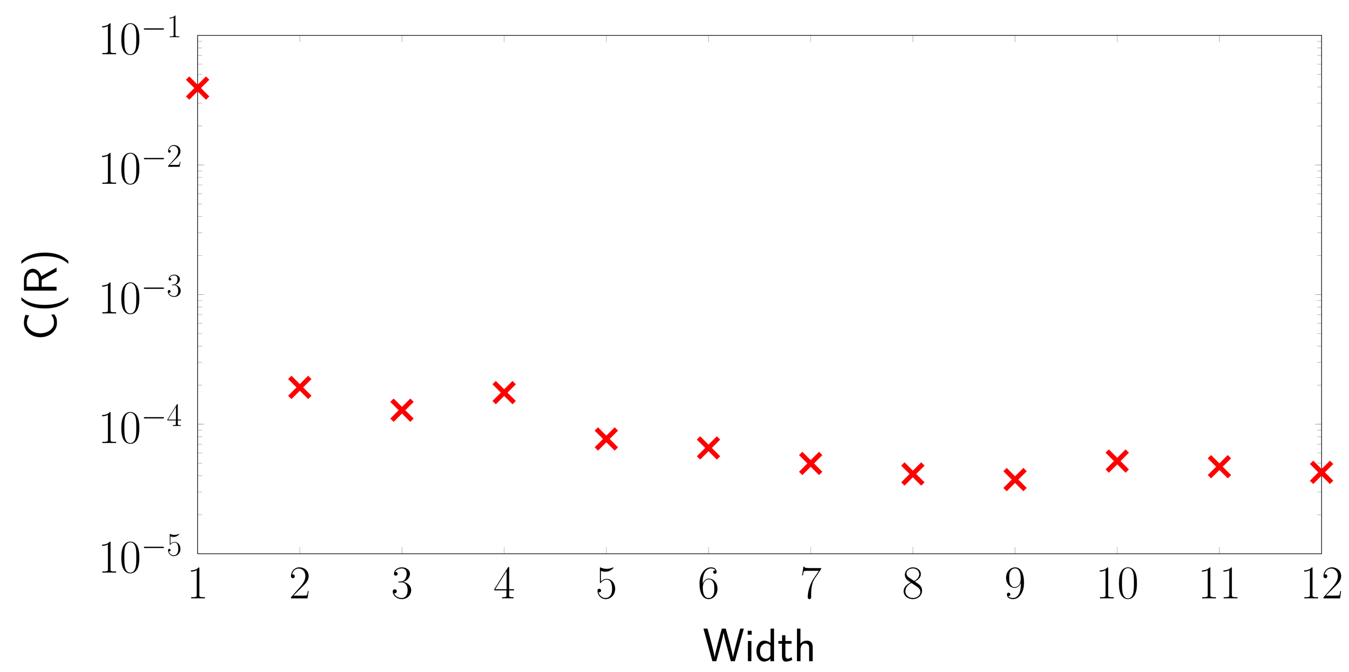

Figure 5: 'Cost' of a ribbon operator in the Honeycomb model as the width is increased. The string operator ($w = 1$) is far from ideal, but extending the support slightly leads to a drastic improvement.