Detecting Topological Order with Ribbon Operators

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Introduction

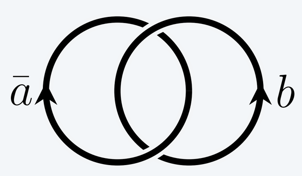
- We introduce a numerical method for identifying topological order in two-dimensional spin models.
  - Topological order away from integrable models.
- Topologically ordered systems interesting for both condensed matter and quantum computing applications.
- Topological entanglement entropy (sometimes) identifies topological order but does not identify which order (which anyon model)[1].
  - Topological entanglement entropy sometimes mistakenly identifys topological order[2]
- By studying 1D operators associated to fundamental excitations (anyons), can efficiently search for topological order.

2D Topologically Ordered Systems

- Long range ordered but no local order parameter.
- Ground space can be used to encode quantum information which is stable under local noise.
- Ground state degeneracy depends on topology of lattice.
- Low lying excitations (anyons) created by string-like operators.
- Phases are robust to local perturbations. Anyon braiding, fusion etc. should be preserved[3].
- Ribbon operators provide a signal of a topological phase, telling us more than just the total quantum dimension.

Properties of Anyons

- Created by string-like operator.
- Localised excitations
  - Undetectable away from particle location.
- Free to move around without changing energy.
  - String operators are deformable.
- Nontrivial braid statistics.

$\mathcal{S}_{ab} = \frac{1}{\mathcal{D}}$ 

- String operators are 'slightly entangling'.
  - Anyons have few degrees of freedom, strings act to entangle them.

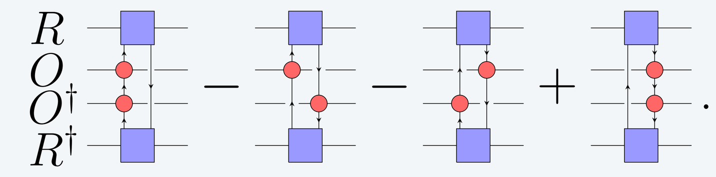
From Anyons to Ribbon Operators

- Operators supported on long strips with bounded width.
  - Proven in LCPC codes[4]
- Away from end points, the ribbon R should commute with the Hamiltonian.
- Existence of a ribbon should only weakly depend on the specific support chosen.
- Commutation of ribbons L and R should capture the topological data (from the $\mathcal{R}$ or $\mathcal{S}$ matrix)
  - We define the twisted commutator $[R,L]_\eta := RL - \eta LR$
- Restrict to the class of matrix product operators.

$$C(R;\eta) := \left\|[R,H]\right\|^2 + \left\|[R,L]_\eta\right\|^2$$

Efficiently Optimising Ribbons

- Translationally invariant matrix product operators are an efficiently representable and manipulable subclass of operators.
- By vectorising the MPO tensors, we can apply standard DMRG to optimise the ribbons.
- Cost function can be encoded as an MPO.

$\left\|[R,O]\right\|^2 =$ 

Analytic Results?

- Method presented here is heuristic.
- Analytic link between approximate symmetries (i.e. operators with small $\left\|[U,H]\right\|$) and ground state degeneracy discussed in poster of Chubb and Flammia.
- Can an analytic extension of this work be used to prove topological order without recourse to ground states?

References

[1] A. Kitaev et al., hep-th/0510092.

[2] L. Zou et al., 1604.06101.

[3] S. Bravyi et al., 1001.0344.

[4] J. Haah et al., 1011.3529.

[5] A. Kitaev et al., 0904.2771.

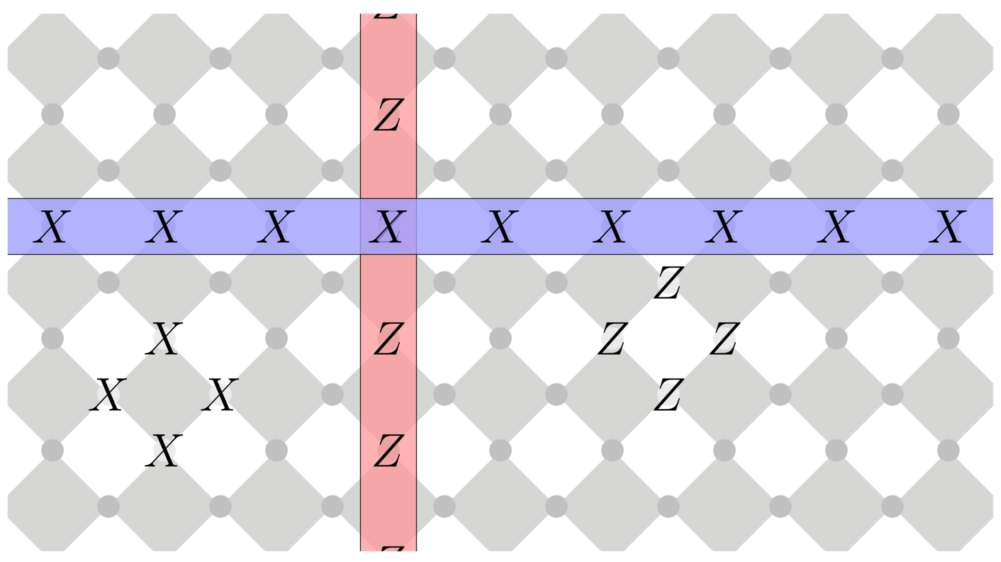
[6] S. Trebst et al., cond-mat/0609048.

[7] A. Kitaev, cond-mat/0506438.

Quantum Double Models

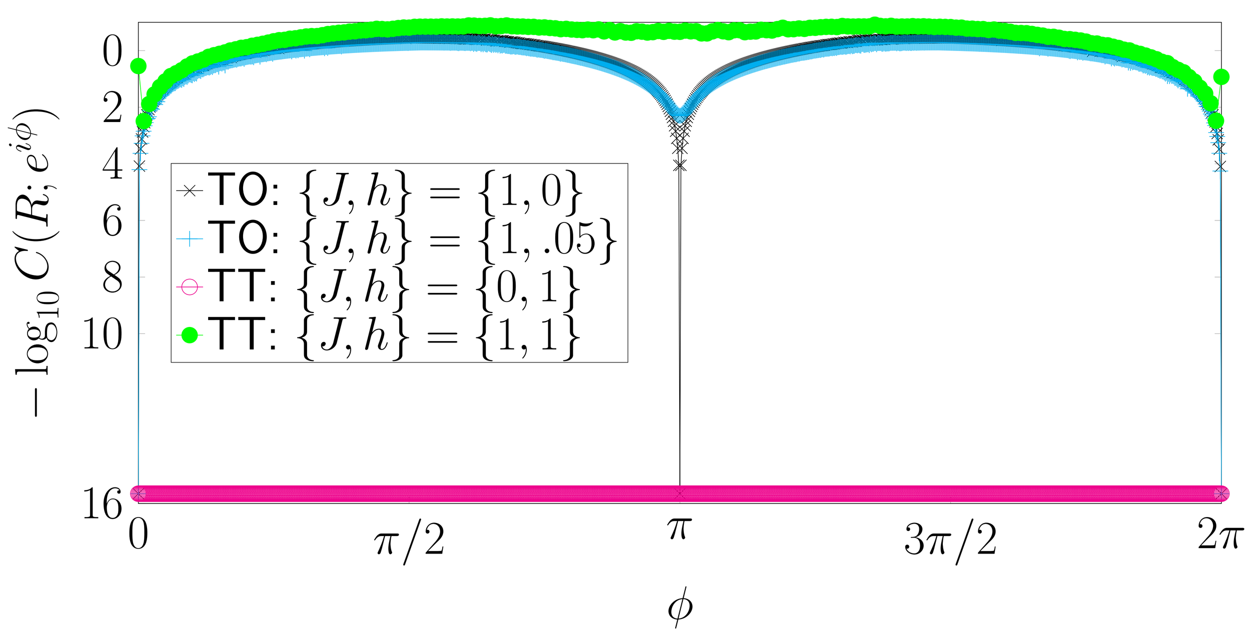
$$H_N = -J\sum_{p\in\{\diamond\}} P_p - J\sum_{p\in\{\diamond\}} Q_p - \frac{h}{2}\sum_j (X_j + X_j^\dagger),$$

$$P_p = \sum_{j=0}^{N-1}\left(X^\dagger \; X \; X^\dagger \; X \right)^k \qquad Q_p = \sum_{j=0}^{N-1}\left(Z \; Z^\dagger \; Z \; Z^\dagger \right)^k$$

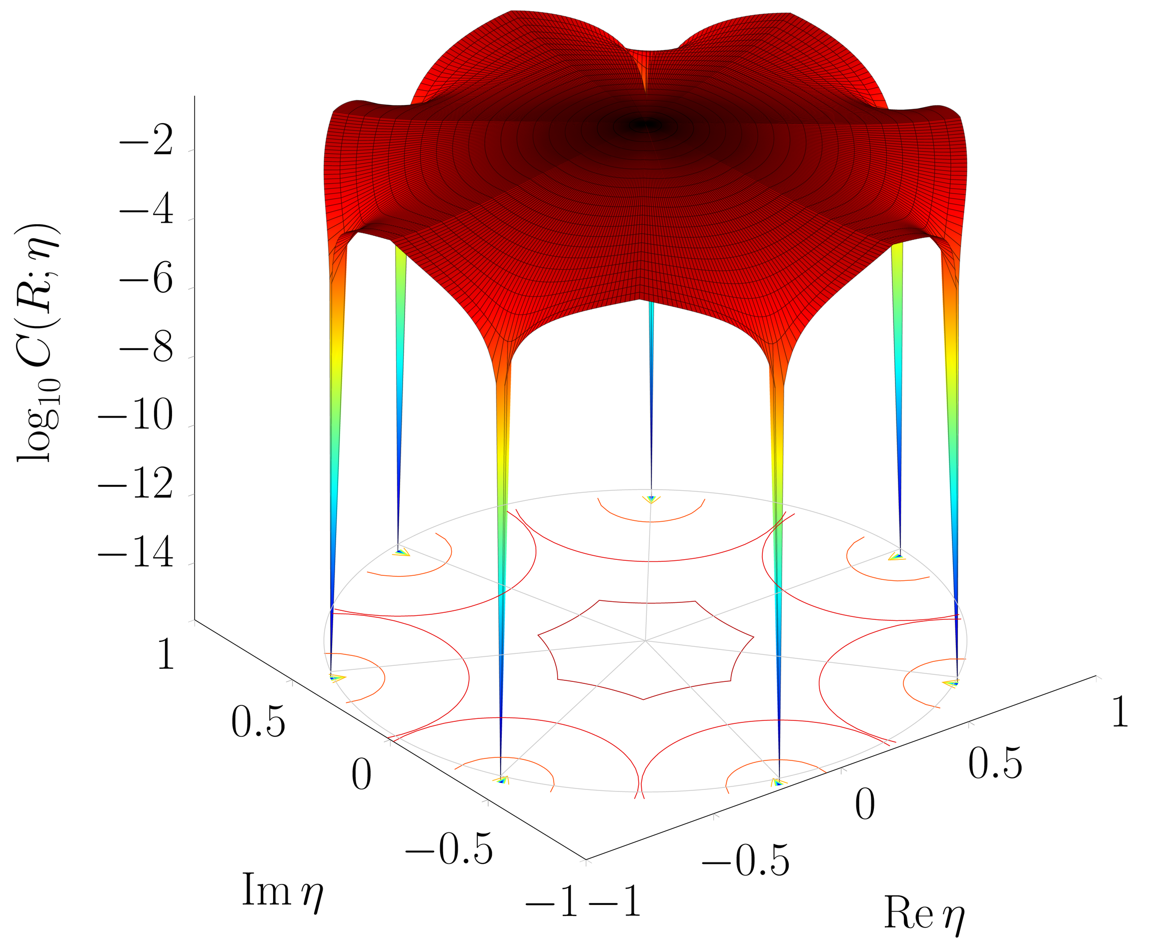


The prototypical ribbon operators are the familiar string operators in the $\mathbb{Z}_N$ quantum double models[5]. These commute with the Hamiltonian but have a twisted commutator with η an Nth root of unity.

- Exactly solvable when $h = 0$
  - Good for benchmarking numerical methods.
- Topological phase.
  - Robust to local perturbations [6]



Cost for pair of ribbons in the $\mathbb{Z}_2$ quantum double model with a Z field. A signal of topological order (TO) is seen in the appropriate phase, no such signal is present in the paramagnetic phase. Similar behaviour is observed in the presence of an Ising-like interaction. (Here $\eta = \exp(i\phi)$)



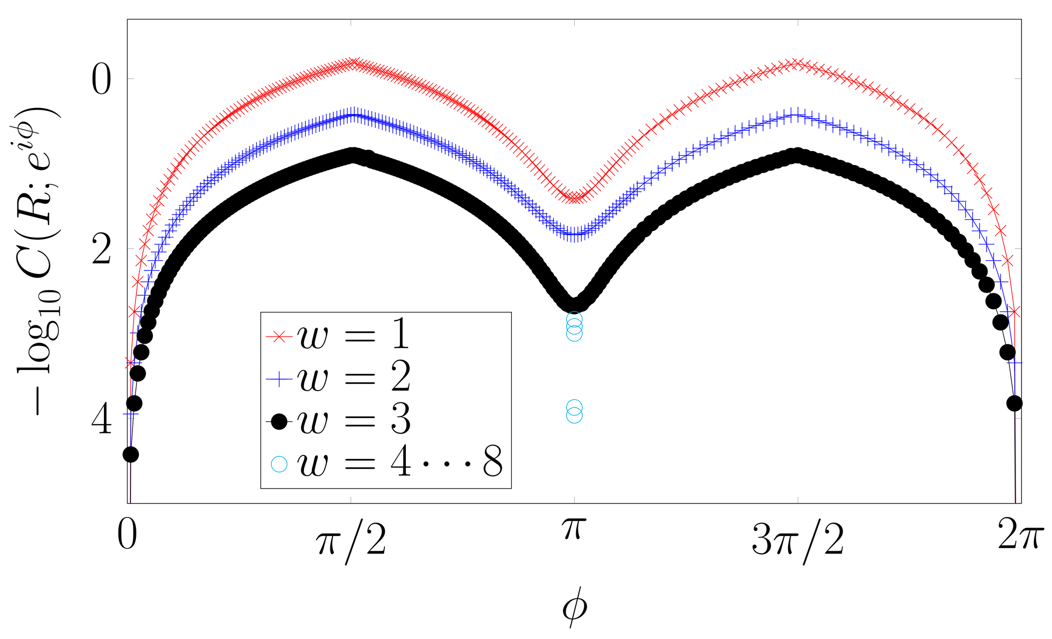
Cost of a numerically obtained ribbon operator in the $\mathbb{Z}_7$ topological phase. The presence of $\mathbb{Z}_7$ anyons is indicated by the strong signal whenever η is a 7th root of unity.

Kitaev's Honeycomb

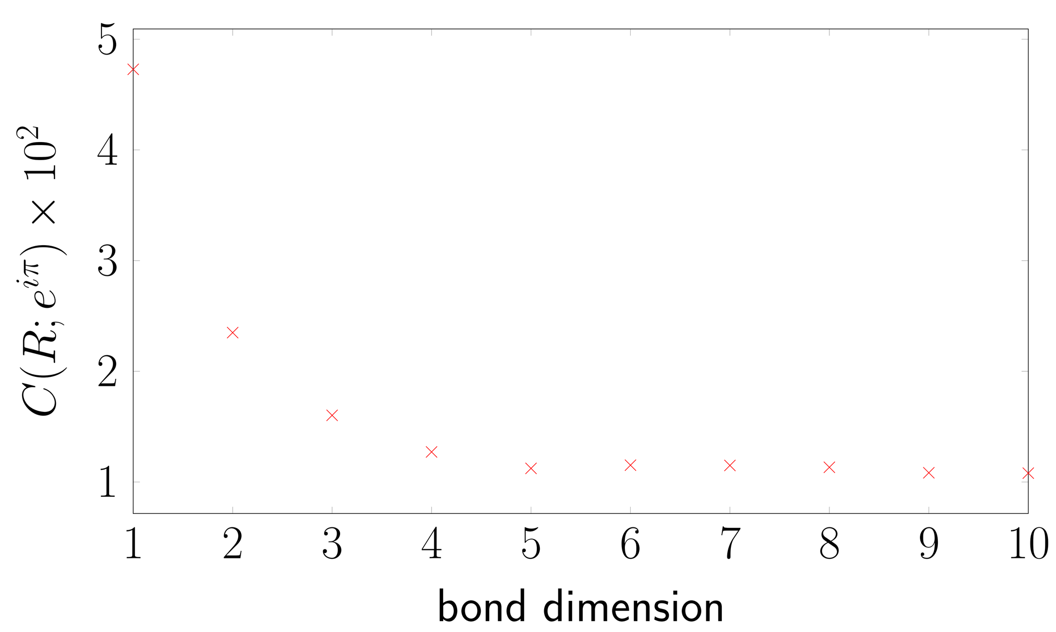
- Spin model on a honeycomb lattice[7]
- Supports a phase with the same TO as the $\mathbb{Z}_2$ quantum double model

$$H = -J_X\sum_{i,j\in X \text{ links}} X_iX_j - J_Y\sum_{i,j\in Y \text{ links}} Y_iY_j - J_Z\sum_{i,j\in Z \text{ links}} Z_iZ_j$$

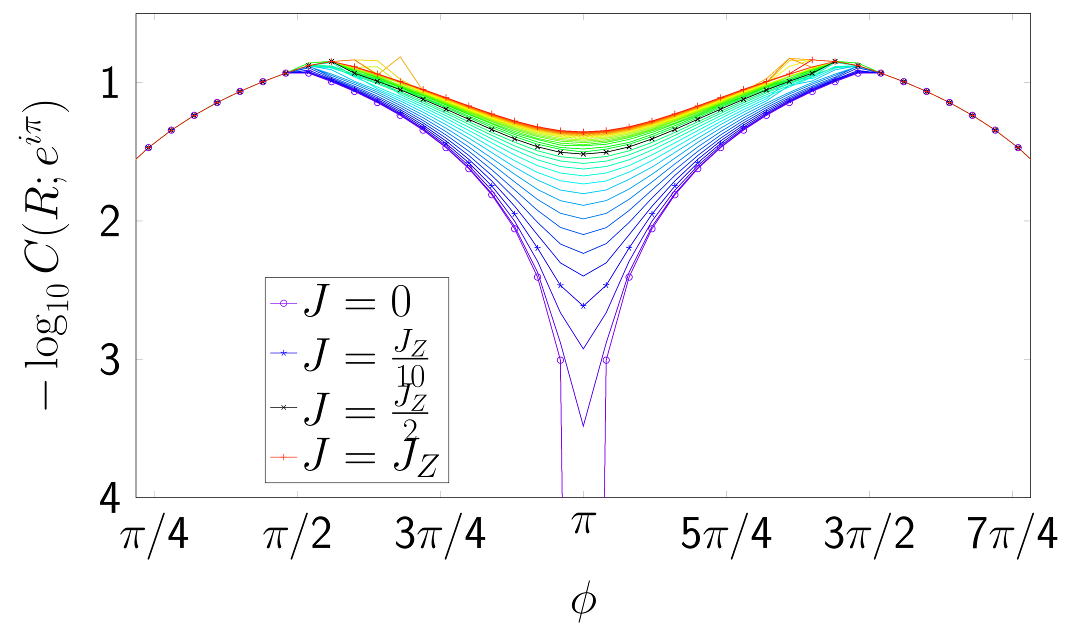
- Perturbatively (around small J_Z) equivalent to the $\mathbb{Z}_2$ quantum double model[7]



As the width of the support is increased, we see the cost decrease.



When increasing the bond dimension, we see the cost quickly saturate. This indicates that we are correct in our assumption that the ribbon operators will be 'slightly entangling'



When the strength of the Z term is increased, the model moves towards a phase transition. We observe the strength of the signal decreases, although does not disappear.